SOME NEW MECHANISMS AND CONCEPTIONS OF STALL
INCLUDING
THE BEHAVIOR OF VANED AND UNVANED DIFFUSERS

BY

STEPHEN J. KLINE

STANFORD UNIVERSITY

PROGRESS REPORT MD-1

TO

THE NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

UNDER

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SUMMARY

The flow in sub-sonic, plane-walled, two-dimensional diffusers with thin inlet boundary layers is discussed. It is shown that only some of the results found in unvaned diffusers and little or none of the results found for vaned diffusers can be rationalized by arguments based solely on the conventional model of the flow employing a two-dimensional boundary layer. It is also shown that some well-known, viscous motions exist in which a steady two-dimensional model of the flow cannot satisfy the laws of motion for the wall layers.

Simple experiments are described which show not only that the inception of stall is a three-dimensional transient phenomenon, at least for the cases observed, but also that under some circumstances, "transitory spots of stall" exist very close to the wall even in extremely mild adverse pressure gradients. A classification of the types of stall is suggested, and a description of the flow mechanisms and important parameters is given for each type.

This discussion of the mechanisms suggests that in order to explain the behavior of stall in all of the observed regimes it is necessary to introduce new flow models in some cases and also to consider not only the forces in the boundary layer but also the effect of continuity and the interaction between the main stream and the viscous portion of the flow. Arguments related to the energy requirements of the boundary layer are also introduced; these confirm the force and continuity arguments

and yield additional information on losses.

Simple experiments that verify the concepts developed are described. The same concepts are also used to analyse other known but previously inexplicable phenomena, and are found to yield qualitatively correct results. Included among the phenomena which can be rationalized by these means are such diverse results as the shape of the pressure recovery curve for simple diffusing passages, the reason for the large pressure pulses and excessive losses found in diffusing passages of low angle, the action of splitters in the wake of bluff bodies, the underfiling of compressor blades, and the detailed action of splitters and multiple vane systems in improving the performance of diffusers.

The work presented is primarily qualitative in nature; this applies both to the discussion of mechanisms and to the rationalization of performance based on the resulting concepts. While certain fragmental mathematical analyses are given, they are used primarily to bolster the arguments rather than as proofs. This is in keeping with the objective of the discussion which is to attempt to clarify, distinguish and explain the basic flow models for the various types of stall. Further work is clearly needed to test the conceptions developed and to perfect the flow models discussed both empirically and theoretically. Despite these limitations the success already obtained in rationalizing performance suggests that this material may already be of some usefulness not only in research but in design.

CONTENTS

LIST OF SYMBOLS

C_{PR} = Actual pressure recovery $= \dfrac{p_2 - p_1}{\frac{1}{2}\rho V_1^{\,2}}$

C_{PR_i} = Ideal pressure recovery $= 1 - (W_1/W_2)^2$

e = Total stored energy per pound of fluid

e_t = Internal thermal energy per pound of fluid (often denoted as u)

H_L = Head loss due to dissipation

$\overline{H}_L$ = Non-dimensional head loss $= H_L/\frac{1}{2}\rho V_1^{\,2}$

h = See Fig. 13a

h_F = Flow enthalpy $= e + pv$

j = See Fig. 13a

p = Static pressure

p_o = Stagnation pressure

Q = Heat

u = Local velocity in the x direction

U = Free stream velocity outside the boundary layer

V = One dimensional velocity of flow

v = Specific volume

W = Width of channel

W_x = Thermodynamic work

w_b = Backflow rate of stalled fluid

w_e = Maximum steady rate at which stalled fluid can be removed

x = Coordinate in direction of wall and direction of flow

y = Coordinate normal to wall

δ = Thickness of viscous layer (See Fig. 5)

ε = Increment in velocity

ρ = Density

τ = Shear stress

Subscripts

$(\ )_1$ = Inlet to diffuser

$(\ )_2$ = Exit of diffuser

$(\ )_w$ = At the wall

INTRODUCTION

This report is one of a series prepared as part of a diffuser research program in progress at the Mechanical Engineering Laboratory of Stanford University. Preparation of this report was begun primarily to explain more fully the results found by Moore and Kline (1) and Cochran and Kline (2) regarding the flow mechanisms and performance of wide-angle, sub-sonic diffusers with and without multiple vane systems. However, as the work progressed, it became more and more evident that the material being developed was of a fundamental nature, and could also be utilized to explain many of the known facts about both stalls and wakes in a wide variety of situations. Since the problem of stall is one of the most important, vexing, and consistently occurring difficulties in both internal and external flow, the studies have been broadened to include additional investigations of several further aspects of stall mechanisms. These studies, which are primarily empirical in nature, have already revealed considerable new information regarding the nature of stall.

This report proceeds by describing the mechanisms of flow found in straight diffusing passages by Moore and Kline (1) and by Cochran and Kline (2). A discussion is then presented which shows that conventional two-dimensional boundary layer theory is inadequate to explain all of results found, and a mathematical proof is given demonstrating that a steady two-dimensional flow model is impossible for the wall layers in a viscous flow under at least some well known flow conditions. Additional tests that were suggested by these results are then described; these additional tests lead to the concept of a new flow model for the inception of stall, and a more detailed picture of the nature of viscous flow in the presence of adverse pressure gradients. These flow models are then utilized to develop qualitative concepts which appear useful in rationalizing a surprising number of known but previously inexplicable results. Some of these results are discussed, and further simple experiments are described which show that the concepts developed do qualitatively predict the behavior of stall.

Since a large number of the matters discussed involve transient effects, still photographs are of limited utility, and primary reliance is therefore placed on descriptions and sketches. Motion pictures illustrating the phenomena discussed are under preparation, as a part of the continuing work, and will be made available to interested groups when completed.

Since the development of these concepts is of very recent origin, only very fragmentary mathematical developments have been carried out to date, and tests of the concepts over wide ranges of Reynolds number and Mach number are not yet available. Final conclusions concerning the ultimate utility of the concepts set forth, therefore, cannot yet be reached. However, the number of applications in which the concepts developed appear to apply is large, and one of the purposes of the present report is the hope that other workers will test the concepts suggested in situations of interest to them and thus extend the data and theory available.

FLOW MECHANISMS IN SIMPLE DIFFUSING PASSAGES

During the course of a systematic study of the behavior of two-dimensional, plane-walled subsonic diffusers over a wide range of conditions, Moore and Kline(1) found that the mechanisms of diffuser flow were considerably different than those envisaged in the classical picture based on the two-dimensional boundary layer. The study revealed that, for a fixed mean velocity profile at the inlet, three parameters apparently were sufficient to fix the flow regime. These three parameters can be taken as: total divergence angle, ratio of wall length to throat width and free stream turbulence. It also appeared from the tests that the flow regime found was independent of Reynolds number based on throat width for all values in excess of that which gave a turbulent boundary layer at or very near the throat of the diffuser. Finally, throat aspect ratio was found to be of no practical significance whatsoever over the entire normal range of practice.

The geometry used by Moore and Kline (1) is shown in Fig. 1.[+] The regimes of flow found are shown in Fig. 2 as a function of the three variables listed above. It should be borne in mind that Fig. 2 applies only to the inlet boundary layer conditions of the unit tested which were held fixed. The names given to the various regimes of flow have been changed from those used by Moore and Kline for reasons that will become evident later. The flow patterns that appear in each flow regime when conventional dye injection techniques are used are also shown by sketches and photos in Fig. 2. If L/W_1 is held constant, and if all inlet conditions including turbulence also are held constant, then Fig. 2 shows that continuously increasing

[+]Further details of the apparatus and procedures are given in reference 1.

divergence angle, starting from zero, causes four regimes of flow to appear in the following order. At the lowest angles, and hence the least adverse pressure gradients, the flow appears as if it were well-behaved and unseparated throughout; this is the zone below line a-a on Fig. 2. It is noted for later reference that in these earlier studies only conventional means of dye injection were employed. Above line a-a of Fig. 2, at somewhat higher angles, large transitory stalls appear. If dye injectors are placed approximately one-quarter of an inch from the wall in the downstream flow, the flow will usually appear to be moving downstream, but now and then will reverse direction and move upstream for a short time. The amount of reverse flow, the size of the reverse motion and the duration of the reverse flow all increase as the angle is increased through this regime. These reverse flows appear first on one wall and then on another, although sometimes they appear more than once on one wall before appearing again on the other. Similarly, the reversed flow was most frequently found in the corners, but was also sometimes found on the middle of a wall. No period of these motions was evident. Thus the reverse flows appeared random in nature.

In this discussion the reverse flows will be referred to as "stalls" or "spots of stall." The entire regime between lines aa and bb on Fig. 2 is therefore called the regime of "large transitory stall." Furthermore, the term "stalled fluid" will be used throughout the discussion to denote any fluid that is moving against the direction of the main flow. In the next paragraph the term "fully-developed stall" will also be introduced. The distinction between a transitory stall and a fully-developed stall is simple. If a given wall area is examined, it is said to have a transitory stall if the flow is forward part of the time and reversed part of the time. If the flow is reversed along the given portion of wall 100% of the time, it is said to have a fully-developed stall.

As divergence angle is increased still further a condition is reached (line b-b, Fig. 2) at which a fully-developed stall appears. In a flow of this type, backward movement is visible all the time on one wall; the flow is relatively steady and two-dimensional, and a large recirculating flow exists in most of the diffuser. The through flow in this regime always proceeds down one wall; the initial choice of wall is random for symmetric geometries, but once located on a given wall the flow remains there.

As will be seen below, still further classification of stalls beyond transitory and fully-developed is necessary to explain the mechanisms found. Also, these two types of stall tend to blend into each other continuously in some cases. However, in most instances the stall is found to be clearly either transitory or fully-developed, and the behavior of the two types of stall is decidedly different. Not only are the flow patterns dissimilar, but, more important, the governing parameters and the concepts needed for rationalization of the mechanisms are in part different for the different regimes.

As divergence angle is increased still further, a point is finally reached, line c-c, Fig. 2, where the through flow moves off the wall and becomes a jet. Fully-developed stalls then appear on both walls. If the angle is subsequently decreased, the flow does not return to the wall when line c-c of Fig . 2 is recrossed; instead it continues as a jet to a lower value as given by line d-d of Fig . 2. Thus an overlap region exists which is indicated by the cross-hatched area in Fig. 2.

Figure 2 also reveals several other pieces of information that will be of importance in the later discussion. The lines a-a and b-b have negative slopes indicating that for a given angle a longer diffuser is more likely to stall. However, the lines c-c and d-d have positive slope indicating that the controlling mechanism is different from that which governs the behavior in the regions of lines a-a and b-b. Figure 2 also shows that, with a fixed mean inlet velocity profile, increasing free stream turbulence raises line b-b very markedly but lowers line a-a slightly on the average.

Moore and Kline (1) also showed that among a considerable number of simple devices tested for possible use in construction of efficient wide-angle diffusers the most promising was multiple short flat vanes. Later thorough tests on such vane systems by Cochran and Kline (2) have verified that such systems are indeed capable of producing wide-angle diffusers of surprisingly good performance, stable flow, and reasonably good exit velocity profiles up to total divergence angles of at least 45° and for a wide range of area ratios. Reference 2 reports the design criteria developed and shows that performance is not critically dependent on precise design or manufacture. The details of this work will not be repeated here, but it is pertinent to note the following. The vanes give best results when they are placed along what are essentially streamlines of the potential flow. The vanes not only have an optimum angle, but also have an optimum length. The optimum angle corresponds to the angle of optimum effectiveness for the individual passages formed, and the optimum length corresponds to the intersection of this angle with line a-a of Fig. 2.

The results just described for vaned and unvaned diffusers were all found for the geometry shown in Fig. 1. However, two sizes of water units and a still larger air unit were employed. In all cases the inlet boundary layers were turbulent and relatively thin. The actual inlet profiles for the air unit are given are given in reference 2. The overall flow patterns found in the three units corresponded completely in all major regards. One discrepancy of a major nature between the water and air units was for a time thought to exist; however, on reexamination of the water table flow it was found that the phenomenon had simply been overlooked in the first tests. The repeatability in the water table was of the order of two degrees and the same results were found by several observers independently. The transient three-dimensional nature of the large transitory stall regime was consistently confirmed by tuft observation in the air unit.

In addition to these data, the air tests also revealed that something was amiss in the region below line a-a of Fig.2 which initially had been called the regime of "no stall" by Moore and Kline. In particular, it was found that the manometers fluctuated far more in the region below line a-a than in the region between lines b-b and c-c in Fig. 2; this was altogether unexpected. However, a check revealed that similar pulsations had been found in diffusers of small divergence angles by many other observers. For example, it was reported to the author privately by both R. T. Knapp and B. Perry that the diffuser in the large water tunnel at CIT had been found to be the source of considerable pressure pulsations despite its total divergence angle of six degrees and very conservative design.

This led Moore and Kline (1) to speculate that perhaps small disturbances of the nature of transitory stall were occurring in some of this region, but that the frequency was too high for visual observation. The existence of such spots of stall at relatively low divergence angles would not be in agreement with the accepted picture of stall as a two-dimensional phenomena that appeared only when relatively large pressure gradients were present. However, as time progressed, more and more evidence accumulated which indicated that it might be necessary to abandon the two-dimensional concept of stall inception in order to explain the known phenomena. Not the least of these evidences were the consistent inability to fully explain the excellent results found with vane systems over a wide range of conditions, and the fact that the stall first appeared as a transient three-dimensional phenomena in every case tested.

In addition, to these data Cochran also compiled a series of pictures of the flow patterns taken from tuft observations in the air unit, and correlated these pictures with the flow performance found. This set of pictures is reproduced in Fig. 3 and shown correlated to pressure recovery in Fig. 4. It is quite evident from Figs. 3 and 4 that the terms "stalled" and "unstalled" are relatively meaningless in this situation. At least for the region of angles above that of maximum recovery (point 2, Fig. 4) the data show that stall is a progression or spectrum of states, and is not a single phenomenon at all.

In late 1956 it was also pointed out to the author by H. Emmons that the explanation of the positive slope of the lines c-c and d-d of Fig. 2 probably did not lie in boundary layer theory, but instead had to be based on arguments related to the stability of the flow pattern.+

As a result of these anomalies, a reconsideration of the entire matter of flow mechanisms was initiated in December 1956. Some results of these considerations are given below.

Since all of the work done in the Stanford University program had been carried out at low Mach numbers, conversations were also held with Mr. R. Scherrer and Mr. J. Lundell of Ames Aeronautical Laboratory of the NACA regarding tests they had been performing on diffusers at higher Mach numbers. It was then revealed that Mr. Scherrer and Mr. Lundell had found results that corresponded very closely to those found by Moore and Kline for the transitory stall

+The author is greatly indebted to Professor Emmons for pointing up the need for stability considerations which are important in several aspects of the problems considered.

regime in conical diffusers with total included angles in the range of 2 - 13 1/2 degrees and inlet Mach numbers up to unity. The techniques and apparatus used in these tests were entirely different from those used by Moore and Kline (1). Furthermore, Mr. Scherrer and Mr. Lundell had independently reached the conclusion that stall inception was a three-dimensional transient phenomena in the cases they were observing. This, or course, provides a highly useful independent check on the observations reported by Moore and Kline (1) and on some of the conclusions reached below. It also suggests that the mechanisms discussed are not confined solely to the Reynolds numbers or Mach numbers occurring in the Stanford tests.

RE-EXAMINATION OF THE FLOW MODEL OF STALL

Let us first examine in more detail why the classical concepts are not adequate for the task of explaining known diffuser performance. Several older concepts are available; included among these are the notions of: (i) flow guidance; (ii) use of individual passages that have small enough angles; (iii) effect of pressure gradient on boundary layer back flow; and/or tolerable pressure gradients. In order to avoid any misunderstanding it should be noted that all of these concepts are useful. Without these ideas, developed at the expense of much effort by previous workers, the work of references (1) and (2) could not have been done, and the present discussion would not exist. However, as will be seen, these concepts in themselves are not sufficient to explain all the known facts in an adequate manner.

"Flow guidance" was the concept employed by K. Frey (3) in designing very short vane systems some years ago. Frey's ideas worked well, but this is virtually all that can be said. "Guidance" as a design concept is very general and loose, and there is no way in which it can be related to the forces in flow field or used to predict results of a given design accurately. It gives some feel for the problem, but it does not give the specific numerical criteria that are so vital to rational design, nor does it give any detailed information about flow mechanisms.

The use of splitters to produce individual passages of small angle was first advocated by K. Oswatitsch many years ago. Oswatitsch noted that for the common range of practice seven degree diffusers performed well, and consequently he reasoned that use of many seven degree diffusers in parallel would produce a good diffuser of wide angle. This notion has never been used very extensively because it only works in some cases. The data of Moore and Kline (1) show that Oswatitsch's notion failed because in reality, the flow conditions are NOT just a function of divergence angle alone. In particular, reference (1) shows the flow regime obtained is also a strong function of the ratio of wall length to throat width, and in some regards a strong function of inlet free stream turbulence as well.

From simple geometrical considerations, it is readily seen that Oswatitsch's suggestion leads not only to a reduction in angle, but also to a large increase in the ratio of wall length to throat width of the individual passages. Reference to Fig. 2 shows that while Oswatitsch's suggestion usually leads to a net gain, the effect of the increase in L/W_1 largely offsets the gain obtained from the decrease in divergence angle. That is, if dividers are extended for the full length of the passage as suggested by Oswatitsch, in many cases an undesirable flow occurs in which one passage stalls out entirely due to the excessive L/W_1 value, and the flow from this passage spills over into the next passage. The recoveries obtained in such cases are poor and the flow sometimes pulsates very greatly. However, the recovery is NOT as poor as would be obtained with no vanes at all but with the same outer walls.

Recognition of the importance of the L/W_1 parameter, coupled with the fact that the vanes were acting as guidance in the very general sense suggested by Frey, led to the idea that the optimum effect would probably be obtained if the passage between each vane was made to include an angle of optimum effectiveness, and also each vane was of the maximum length for which little or no stall would occur in the passages based on the data of Fig. 2. That is, the individual passage angles should be around seven degrees and the length of each vane should be adjusted so that the value of L/W_1 for each passage lies just on or below the line a-a representing significant stall on the flow mechanism plot of Fig. 2. This suggestion has proven to be entirely correct. In the cases reported in reference 2, the optimum recovery has been found in every instance to lie at or very near this condition. Additional gain can also be realized by proper placement of the vane cluster in the unit as a whole. This is apparently due to two facts: (i) it is not desirable to accelerate the flow more than necessary at the throat, and (ii) the outer walls already have boundary layers at the section where the vanes begin and are thus more liable to stall.

From the overall point of view the foregoing discussion of diffuser design not only appears quite satisfactory, but it also produces very good wide-angle diffusers. How-if one begins to examine the details more closely, some discrepancies immediately appear in the explanations given. In the first place, the concept of "each passage is such that it will not stall" is an overall concept that does not lend itself to explanation of the details of the flow, nor does this overall concept give any basic reason for the good performance uniformly obtained. In the second place, if one examines the details of the flow in terms of the usual modern concepts, that is, in terms of pressure gradients and steady two-dimensional boundary layer theory, some of the results are apparent, but others are quite inexplicable.

To begin with, let us remind ourselves of the accepted notions of how a positive pressure gradient brings on a stall in a boundary layer. Since the pressure forces oppose the motion, it is reasoned that the boundary layer near the wall which has a large deficiency of kinetic energy is brought to rest by the pressure forces. Somewhat downstream from the section where this fluid comes to rest, the pressure gradient is adverse and it is presumed that this causes the entire flow near the wall to reverse direction. Separation of the whole mainstream from the wall then results.

Using the flow model just described, many attempts have been made to correlate the various results for the turbulent boundary layer using theory, empirical data and combinations thereof. The most notable recent attempts include the well known method of Tetervin and Doenhoff (4), Ross and Robertson (5) and F. Clauser (6). Unfortunately, none of these numerous attempts have given rise to a method that seems to predict separation adequately for all cases although the methods of references (5) and (6) respectively seem to come much closer to an understanding of the parameters involved and to reliable predictions than the older methods.

It is not the purpose of the present discussion to summarize the results or methods of sucn work. However, it is pertinent to the present discussion to note that a great deal of effort by many exceptionally able workers has gone into the development of such methods in the hope of providing reliable means for predicting stall of a turbulent boundary layer in a positive pressure gradient, and that none of these efforts to date have been totally successful. It is also relevant to the following discussion to note that in all of these analyses the assumption is made that the flow is steady in the mean and two-dimensional. Furthermore, in all of these analyses, qualitative agreement is reached on two points: (i) stall is governed entirely by initial boundary layer condition and by the applied pressure distribution of the potential flow, (ii) other things being equal, greater pressure gradients and longer application of these gradients both uniformly tend to increase the likelihood of stall. These results can and have been put into many forms, but there is no disagreement in any of the modern theory on the correctness of the qualitative results (i) and (ii) above.

If the qualitative results just cited are applied to the results found by Moore and Kline (1) and Cochran (2) on vanes, a difficulty immediately arises. At first it was thought that the action of the vanes could be explained by noting that the pressure gradient was improved in the region just downstream of the throat at the curved walls. This region is the worst positive pressure gradient in the entire unit and hence is critical. However, the ability to increase the angle at which good flow can be obtained using vanes by a factor of more than three would make this explanation very dubious. Furthermore, the truly spurious nature of this argument can be seen in the following way:+ Consider a diffuser with a total divergence angle of 18 degrees an L/W_1 of 8 and a very low entering turbulence level. To make the matter clear, let us also assume that the unit has precisely the same geometry as that studied by Moore and Kline (1). If the unit has no vanes or other boundary layer control, the data shown in Fig. 2 indicate that the unit would operate in the fully-developed stall region in which very poor performance is obtained; for the geometry tested by Moore and Kline, the recovery would be twenty-five percent or less of the ideal recovery. Now if one inserts a splitter vane running the full length of the unit, following the ideas of Oswatitsch, each passage would have a total divergence angle of 9° and an L/W_1 of 16 . Thus Fig. 2 shows each passage would be well up into the large transitory stall regime, and the flow would be extremely unsteady with a large back flow in one of the two channels. Nevertheless, the recovery would be somewhat improved. Lastly, suppose that a single somewhat shorter vane was inserted along the centerline of the unit according to the design criteria set forth in reference 2. The vane would then start somewhat downstream of the throat and would end short of the exit to yield an L/W_1 of eleven or twelve. This unit would not give optimum performance

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+ The author is indebted to F. Clauser for clearly pointing up this paradox.

(for eighteen degrees, two vanes would give slightly better performance than one vane), but it would definitely give a recovery 50 or 100% greater than either of the other units, and would have a much smoother flow, and more uniform exit velocity profile.

If we try to correlate these facts with the accepted notions about boundary layer stall, a contradiction occurs. If we had used two or three vanes, then it could have been argued that appreciable pressure changes would have resulted in the most critical region just downstream from the throat on the curved walls. But it is quite obvious that if the flow is two-dimensional and steady, insertion of a single thin vane along the centerline does not alter the pressure distribution of the potential flow appreciably. (A very thin vane does not alter it at all and the real value is 1/16 inch thick compared to a minimum channel width in which it occurs of 3-1/2 to 4 inches). Thus we find a paradox in the sense that two-dimensional boundary layer theory suggests that the separation depends only on the magnitude of the potential flow pressure gradient and the duration for which it is applied for fixed inlet conditions, but the data of Cochran and Kline (2) show that by insertion of vanes that make little or no changes in the pressure distribution of the potential flow tremendous changes in performance and the entire flow pattern actually are brought about.

These results have now been thoroughly verified. There can no longer be any reasonable doubt that the results cited are correct in the light of the uniformly good performance found by Cochran and Kline (2) with vanes. The only reasonable conclusion that can be drawn is that something is missing in the two-dimensional boundary layer theory. To find a hint concerning what that something is, we turn back again to the mechanisms of the flow since it is well known that when one finds a gross failure of an entire body of theory, particularly in viscous flow, the most likely point for the difficulty to lie is in the flow model employed.

At this point in the work, a careful study of the usual arguments relating to the forces which are involved in maintaining the flow of a boundary layer in an adverse pressure gradient was made. Several cases were found in which the numbers obtained from diffuser data could not provide a two-dimensional force balance for the layers very near the wall. Somewhat later a more general and clearer argument that does not depend on any special data whatsoever was found. Since this later argument is clearer and more general it is reproduced here.

Consider the control surface shown in Fig. 5. Point "m" may be either the throat of a diffuser or a point of minimum pressure on an airfoil. The momentum equation of the two-dimensional boundary layer for incompressible flow can be written:

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} \qquad (1)$$

Since the value of δ is arbitrary, we can select it such that at the outer edge of the control surface the inertia stress $\frac{1}{2}\rho u_L^2$ is small compared with τ_o the wall shear at point m. Since the velocity u is zero at the wall, such a δ always exists. The balance of forces on the small element in the x direction is then

$$(\tau_L - \tau_w)\cancel{dx} - \delta \frac{\partial p}{\partial x}\cancel{dx} = 0.$$

However: equation (1) at the wall gives:

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} \qquad (2)$$

Eqn. (2) requires that boundary layer velocity profiles have the general shape shown in Fig. 6. Furthermore: for

$\frac{\partial p}{\partial x} \leq 0$; $\tau_w > \tau_L$ and for $\frac{\partial p}{\partial x} > 0$ $\tau_w < \tau_L$.

At point m, Fig. 5, the boundary layer has a profile such that $\tau_w > \tau_L$ since it has been generated in a negative pressure gradient, but somewhat farther downstream it must alter such that $\tau_w < \tau_L$. If we plot the shear values versus x, they must therefore appear as shown in Fig. 7. Since the distance dx is arbitrary, we can now define it to be the dx shown in Fig. 7. It therefore follows that we can define a positive quantity β such that it represents the average total shear on the control surface over the length dx as follows:

$$\beta \triangleq \overline{\tau}_w - \overline{\tau}_L > 0$$

Then the x balance of forces becomes

$$-\beta - \delta \frac{\partial p}{\partial x} \neq 0 \qquad (3)$$

Equation 3 shows, that under the assumptions used, a balance of forces is impossible inasmuch as the only two forces present act in the same direction . The only assumption employed is that of steady two-dimensional flow, and it is therefore apparent that a continuous wall layer cannot exist under the conditions analysed with steady two-dimensional flow. This suggests that strong transitory stall probably will be found at and just downstream of a minimum pressure point, and the observations show that this is indeed the case.

In addition to these results, several other workers, most notably Kalinske (8), have shown that the losses in apparently unseparated diffusers were larger than could be accounted for by wall shear and turbulent decay. These results were again confirmed by the data of Cochran and Kline (2). In fact, even if one assumes that the wall shear is that which would occur for zero pressure gradient, which is far too large, all the losses cannot be accounted for even at relatively small divergence angles. Since it was also known that pressure pulses were found at relatively low angles in diffusing passages, and since it had been shown that stall inception appeared as transitory "spots of stall" in the work of Moore and Kline (1), it was hypothesized that these spots of stall might occur very near the wall even at very small angles of divergence, that is, for very mild adverse pressure gradients. In particular, the information available suggested the following:

a. Spots of stall may occur even in the presence of very mild adverse pressure gradients;

b. These spots originate very near the wall, probably in the laminar sublayer;

c. The size, number and upstream movement of the spots all grow with an increase in the magnitude of the adverse pressure gradient;

d. The size, number and upstream movement of spots of stall all grow with increase in length over which the pressure gradient is applied;

e. The mechanism by which such spots are carried back downstream is by leaving the wall and being swept away by the momentum of the faster moving layers;

f. Other things being equal, thicker inlet boundary layers will yield more spots of stall.

At this time, Mr. J. A. Miller[+] had already developed a technique of flow visualization which utilized dye injected through a six inch length of small hypodermic tubing sweated onto a standard medical hypodermic apparatus (see Fig . 8). This apparatus provides a means for locating dye accurately in a water table flow without disrupting the flow too greatly. As soon as the conclusions set forth above were reached, Mr. Miller applied this technique at the wall and immediately he found the small transitory stalls predicted. Since that time the technique has been improved, and the experiments repeated a large number of times. A considerable amount of information about this mechanism is now available. While many details are still under study,

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[+] Research Assistant, Stanford University.

enough is now known to complete a good portion of the qualitative picture. Some of the more critical tests are described in the following section.

SMALL TRANSITORY STALL USING DYE INJECTION

EXPERIMENTAL TECHNIQUE

The following technique has been employed in the studies reported below on the action of the wall layers. Using the hypodermic tubing and hypodermic shown in Fig. 8, a straight line of dye is placed on the wall normal to the flow. Any dye accidentally located in the outer layers washes downstream very rapidly, but at the low velocities (0.2 - 2.0 ft/sec) employed in the water table, the dye hangs on the wall for several minutes giving a remarkably clear picture of the wall layers.

These studies confirm that all of items a, b, c, e, and f hypothesized above are true.+ Item d has not yet been tested independently since this will require the construction of a new apparatus.

RESULTS AT LOW DIVERGENCE ANGLES

The actual tests have shown that transitory spots of stall do exist under some conditions for all adverse pressure gradient flows; in some cases this includes total divergence angles as low as one and two degrees. In such flows, normal dye injection into the channel shows a pattern that looks unstalled, but careful inspection of the wall layers reveals intermittent spots of stall at every point on the wall. In very low adverse pressure gradient flows, a reverse flow is not found on every trial, because the frequency of occurrence is low. However, two or three tries were sufficient to show up at least one clear case of a spot of stall in every instance in the whole first series of tests.

As was expected, the number of spots of stall and their size both increase as the pressure gradient is increased. At very low pressure gradients normally only one spot of stall appears at a time at a given section on the wall. The normal water depth employed in these tests is 4 inches.

+Studies of the characteristic periods of these spots of stall are under way, but no conclusions are yet available.

With this depth, a high inlet turbulence level, a total divergence angle of 2 degrees and an L/W of 8, one spot, perhaps a 1/16th of an inch in diameter, might appear in a minute or two at a given location. No systematic size and frequency counts have yet been made. Such a spot may move upstream anywhere from a very small amount up to an inch (average upstream movement is perhaps 1/4") before leaving the wall. As soon as the spot of stall leaves the wall it is caught up by the mainstream and disappears downstream very rapidly. This sequence of events is shown in Fig. 9. The forces that cause the spot of stall to leave the wall are not yet fully understood, but they are probably related to an interaction with some oncoming flow.

With the same water depth, high inlet free stream turbulence level, and $L/W_1 = 8$, but with a total divergence angle of twelve degrees, several spots of stall often appear at once on a given cross section of the wall; they may move upstream as much as several inches before being washed back downstream, and the rate of occurrence is much larger. Spots of stall have been found almost every time dye was injected on the wall under these conditions. A typical configuration is shown in Fig. 10.

As would be expected, spots of stall occur more frequently in the corners than along the main portion of the wall. Also, as would be predicted from the analysis of forces on a minimum pressure point given above, very strong spots of stall are found just downstream of the throat. In fact, the transitory stalls at this location have always been the strongest found. This was noted before the analysis given above was available, and it has been recorded independently by several observers including Mr. Scherrer and Mr. Lundell working at higher Mach Numbers and Reynolds numbers on another apparatus.

Indeed, under these conditions the transitory stalls are so strong at the throat, that frequently four or five strata of flow are seen in the wall layers with each successive strata moving in opposite directions along the wall. The stalled strata peel off intermittently, so that the action is transient rather than steady with the flow at any given point going first one way and then the other in an apparently random fashion. This "layering" near the throat is so strong very close to the wall that sometimes a low speed vortex is formed flat against the wall.

At this point, it is well to describe further what is meant by "spot of stall" and also to indicate the tests that were made to insure that the phenomena observed were not artificially created by the dye injection technique employed.

The term "spot" has been adopted from the work of Emmons, Schubauer and Klebanoff, et al on transition. Actually there are some differences and some similarities to that application, and it might be better to use another term. However, the word seems appropriate, and there should be no cause for confusion between a "spot of stall" and a "spot of turbulence." In the connotation used here, a spot of stall is simply an isolated segment of fluid, that is moving upstream rather than downstream. In the cases thus far observed the spots are transitory in nature and do not maintain any given position in space for long periods of time. Instead they appear first in one location and then in another in an apparently random fashion. Thus the regime called transitory stall on Fig. 2 consists of a main flow moving downstream over the entire channel cross section but with areas of temporary local backflows or "spots of stall" occurring . These observations will be correlated with performance below.

In regard to the matter of flow distortion due to injection many facts indicate that this is not the source of the spots of stall observed. First, simultaneous observations in zones of negative pressure gradient under the flow conditions just described above do not show the backflows observed in regions of positive pressure gradient. That is, if precisely the same technique is applied ahead of the throat of the diffuser, no spots of stall are observed, and the whole nature of the flow in the wall layers appears distinctly evener. Observations on an airfoil similarly reveal isolated spots of stall downstream of the point of minimum pressure even for very low angles of attack, but do not reveal such spots of stall in the zone of negative pressure gradient. Furthermore, careful tests have been made in which the angle of dye injection has been made such that any momentum created by injection would be in the downstream direction by slanting the hypodermic tube at a very small angle to the wall pointing downstream. Under these conditions, the same phenomena are observed; it is not possible to see any change by simple observation. Furthermore, in the tests made, frequently a single layer is observed to move upstream then downstream and then upstream again several times following one injection of dye. Such a motion could be attributed to an initial disturbance only very, very rarely.

It can therefore be concluded from these tests that the hypotheses set forth above have been well confirmed (with the exception of the effect of length) for at least one set of conditions. Since the search for this mechanism was begun in order to find a mechanism that could supply the forces needed to move the wall layers downstream against an adverse pressure gradient in certain special cases, it seems logical to conclude that transitory stall is a type of self-induced augmented mixing that can create the forces needed for the wall layers to move downstream in a positive pressure gradient. Since it removes fluid of very low energy from near the wall and replaces it with fluid of higher energy, it can also be viewed as a self-energizing of the boundary layer.

At this point in the actual work, the results just cited were utilized to rationalize many of the known results on diffusers and in other situations. After this had been done, it became apparent that several open questions remained, however. These included the following: (i) "How do spots of stall originate?" (ii) "How does large transitory stall break down into fully-developed stall?" (iii) "What are the parameters governing the growth of a spot of stall?" (iv) "What are the parameters governing performance in the region of fully-developed stall?" and (v) "What are the parameters governing the change in flow regime from fully-developed stall to jet flow?" In the process of considering question (v) another question also arose; this is: "What determines whether a fully-developed stall will be truly steady in nature or instead will periodically shed wake fluid?" This question is of considerable importance not only in diffusers but also in connection with bluff bodies, with sudden enlargements and contractions, with compression corners and in numerous other applications.

Simple water table tests that shed at least some light on these questions have now been completed, and from the viewpoint of clarity it is better to discuss these experiments next, and to delay rationalization of other known results. Although in the actual work, the two went hand in hand.

FURTHER WATER TABLE TESTS ON MECHANISMS OF STALL

CLASSIFICATION OF STALLS

In order to comprehend the tests to be described below, it is helpful to look again at Fig. 4 which shows the regimes of flow correlated with the performance of a simple plane-walled diffuser. The reason for the name of the zone which lies at angles below that of optimum recovery (to the left of point 2) is now clear. The tests just described confirm the fact that some stall may exist in diffusers over the entire range of angles shown, but the average cross-sectional area stalled tends to increase as the angle is increased.

At the very low angles, the amount of stall is zero or very small. If it occurs, one finds very small isolated spots that remain stalled only for short periods of time. The amount of stall tends to increase gradually until the point of optimum recovery is reached. At this point (point 2, Fig. 4) the rate of increase in head loss as a function of angle equals the rate of increase of ideal recovery with angle as shown in reference 2 and transitory stall has always been found at these angles. Thus it is not surprising to find that while the stalls are still isolated in nature they are becoming relatively large. Hence, the point of optimum recovery for a single passage should correspond very roughly to the line a-a on Fig. 2, and it does for the data thus far checked. The true nature of line a-a of Fig. 2 is thus seen to be not "first stall" as originally thought, but instead is a line of "first large stall." This is quite in accord with the way in which this line was found.

In the earlier observations dye was injected upstream of the throat against the wall and also downstream by means of movable injectors. Neither of these means shows the wall layers clearly for the following reasons. Injection upstream on the wall was the means used to visualize the wall flow, but it has two serious deficiencies for observation of small transitory stall (which, of course, was not known to exist when the first set-up was devised). In the first place, it shows the boundary layer at only one level of the four inch depth. Since the transitory stall is three dimensional and usually appears in small spots, most of the spots would inevitably be missed. In addition, and even more serious, the dye injected upstream of the throat does not really show the wall layers downstream at all; it shows instead the portion of the boundary layer somewhat farther from the wall. The reason for this is that most of fluid that starts against the wall upstream is deflected outward by the displacement action of the very slow moving wall layers downstream. Consequently, there is never enough diffusion of dye into the wall layers to make them visible, and they are thin enough so that this lack of visualization is very hard to perceive by eye. This fact was demonstrated very clearly by employing a conventional upstream injector simultaneously with dye injection of the sort used in the later tests. The dye injected onto the wall lay completely inside all visible dye coming from the upstream injector. Thus upstream dye injection against the wall misses small transitory stall in the wall layers entirely.

The dye injected from movable probes downstream, was even less apt to show small transitory stall since the injectors were never brought too close to the wall for fear of artificially disrupting the boundary layer. Reliance was placed on the upstream wall injectors to show the boundary layers, but as just explained they miss small transitory stall. Since the movable downstream dye injectors were moved back and forth carefully in the early tests, they did locate three-dimensional stalls, but only after the stalls had reached a size large enough to extend out a quarter of an inch or more from the wall. Thus line a-a of Fig. 2 is really a line of "first large transitory stall". Consequently, it now seems more appropriate to call the region under line a-a of Fig. 2 and to the left of point 2 in Fig. 4 the zone of "small transitory stall."

As shown by Fig. 4, at angles somewhat greater than that for optimum recovery, (points 2 to 3, Fig. 4, between lines a-a and b-b, Fig. 2) the transitory stalls begin to coalesce into large areas of stall. In this region the performance drops rapidly, large flow pulsations occur, and performance is hard to predict; it is a poor region for design. At still larger angles (near point 4, Fig. 4) a destabilization of the whole flow apparently occurs, and fully-developed stall appears on one wall. Finally, the curve of recovery begins to rise again as still further increases in angle occur. (This last rise is probably not general as will be seen later).

In the air unit, angles large enough to cause jet flow to occur spontaneously cannot be obtained. However, even the portion of the curve shown in Fig. 4 is sufficient to show that several changes in flow mechanisms must occur to create the complex shape found. In fact, in retrospect, it is difficult to see why it was ever proposed to explain a curve with at least two extremums and an inflection point with a single change in flow mechanism, that is, in terms of simply stalled and unstalled flow.

With these facts in mind, the questions asked above are clearer. The first question deals with the inception of small spots of stall at very low angles, that is to the left of 2 on Fig. 4. Question (ii) deals with how the instability occurs that brings on fully-developed stall near point 4 in Fig. 4. The third question relates to how the spots of stall increase both as angle is increased and in connection with the instability that causes fully-developed stall. Questions (iv) and (v) deal with flow behavior in the fully-developed stall region, which is quite different from a flow that fills the entire passage, and the final question suggests that there are two kinds of fully-developed stall. Thus the large number of questions arises from the varying aspects of the phenomena of stalls. A further classification of stalls therefore seems in order.

At present it is believed that stalls might logically be classified into four basic types according to the concepts apparently needed to rationalize behavior. These four types include two kinds of transitory and two kinds of fully-developed stall as follows:

I TRANSITORY STALL

A. Small Transitory Stall: Characterized by small isolated spots of stall that do not interact with each other or remain fixed in space. This is the region below curve a-a Fig. 2 and left of point 2 on Fig. 4b.

B. Large Transitory Stall: Characterized by large flow fluctuations interactions of more than one spot of stall and strong interactions between the mainstream and the stall spots. Lies between a-a and b-b of Fig. 2, and between points 2 and 4 in Fig. 4.

II FULLY-DEVELOPED STALL

A. Fully-Developed Steady Stall: Characterized by a large area of wall that has backflow one hundred percent of the time. Little or no wake fluid is shed; it merely recirculates.

B. Fully-Developed Unsteady Stall: Characterized by a large area of wall that has a backflow one-hundred percent of the time, but large segments of wake fluid are periodically cast-off and go downstream.

In this classification both the jet flow regime and the fully-developed stall regime are considered to fall in class II A. The distinction merely illustrates the nature of the stability problem that is associated with this class of flow and is discussed below. Class II B is important in connection with bluff bodies and sharp corners as noted above, but it does not occur in the geometry studied by Moore and Kline (1); the reasons for this will become evident later.

SMALL TRANSITORY STALL REGIME

As noted above this is the region lying below curve a-a Fig. 2 and to the left of point 2 on Fig. 4b. In this region normal dye trace methods always show an apparently unstalled flow, but close observations of the wall layers reveal that small isolated spots of stall are sometimes present even to very low divergence angles. Since the spots are very small and occur individually, a question concerning how they commence naturally arises. The two logical possible sources are perturbations of the flow and wall roughness. In the present tests, the low velocities used gave a hydraulically smooth wall, and it was found that under these conditions unevenness or small bumps on the wall seemed to have little effect. Disturbances from the mainstream, on the other hand, had a pronounced effect. In fact, such disturbances appear to be the sole origin of the spots in the tests to date, and they appear to have a large effect on the size of the spots of stall, the frequency of occurrence, and the minimum divergence angle at which stall spots can clearly be discerned. However, these tests are still quite limited in scope and further tests using carefully controlled mainstream fluctuations will have to be made before the matter is fully understood.

Two types of tests on the inception of stall have been performed in the water table. In both tests a mild adverse pressure gradient was used (total divergence angle of approximately 3° and L/W_1 of 8). In the first test several observations were made of the flow at one point on the wall using the dye injection technique described above. These observations were carried out with a relatively high inlet turbulence level in the water table (see description reference 1). These observations showed only one spot of stall at a time with a diameter averaging perhaps an eighth of an inch. One such spot occurred on the average about every 30 seconds. Then a horizontal grid of 3/8" rods with 3/8" gaps was placed across the flow upstream from the point of observation on the wall to increase the free stream disturbances, and the tests were repeated. Immediately, the number and size of the spots of stall increased. As the disturbance size was increased by moving the grid of rods closer to the point of observation, and thus decreasing the decay of the large scale fluctuations, the spots of stall became still more numerous and larger. When the grid was located only a few inches upstream along the wall under study, as many as ten spots appeared at once on the 4" wall and the average diameter of the spots appeared to be two or three times the size found at the original turbulence level.

The second test involved introduction of disturbances into the flow with the hypodermic needle used for dye injection in order to actually see the impact of the disturbances at the wall. In this test a dye trace was placed on the same location of wall as in the first test and with the original turbulence level, that is, without the 3/8" rods; again only one very small spot of stall was usually visible at a time. Then while the dye on the wall was still visible, the hypodermic tube was oscillated back and forth across the flow a short distance out from the wall and upstream from the location of the dye trace. At the same time as it was oscillated a small amount of dye was injected. In this way the disturbances generated could be seen moving downstream in a pattern much like that of a Mach cone in supersonic flow. Up until the time these disturbances struck the wall with the dye trace, no change in the pattern of spot stall was visible, but as soon as the disturbances reached the dye trace, almost at once six to ten small spots of stall appeared at the wall. Each spot appeared much like a small three-dimensional stagnation point flow.

It is believed that these two experiments show rather clearly that at least one origin of spots of stall is the effect, at the wall, of the pressure pulsations generated by free stream velocity fluctuations. Normally, fluctuations large enough to cause a complete reversal of the free stream flow do not occur, but the following oversimplified analysis suggests that such flow reversals might readily occur at the wall due to the non-linear nature of the interaction between pressure and inertia forces.

In the free stream a small velocity change causes a pressure disturbance given approximately by Euler's equation as:

$dp/\rho = - UdU$, where U is the free stream velocity (4)

If the pressure fluctuation occurs near the wall, we can assume as a first approximation that it propagates to the wall without attenuation. However, at the wall it will then cause a convective velocity alteration of magnitude:

$dp/\rho = - udu$, where u is some average velocity of the wall layers. If we equate the two pressure disturbances according to the assumption of little attenuation, then as a first approximation

$$du/dU = U/u. \qquad (5)$$

Equation (5) is a crude approximation because it not only neglects the effect of viscosity and attenuation of the fluctuation in pressure, but it also assumes that the pressure fluctuation is applied steadily to some small stream tube. In actuality, the pressure fluctuation is not applied steadily, and the time dependent term of Euler's equation is important. Nevertheless, equation (5) shows the terms that might readily bring about a small flow reversal in the wall layers.

The data on turbulent boundary layers, as given for example by Klebanoff and Diehl (8), show that the turbulence intensity near the knee of the velocity profile is of the order of ten percent. The velocity at this location is of the order of fifty to seventy percent of free stream velocity, and thus flucutations of the order of five percent of free stream velocity are normally present very near the wall in a turbulent boundary layer. Hence it is relatively easy to envisage small flow reversals in the layers very near the wall particularly when the pressure forces are such that they tend to sustain motions in an upstream direction.

The foregoing analysis suggests that flow reversals may also occur very near the wall in laminar boundary layers if strong fluctuations are present in the free stream, and may even occur in the negative pressure gradient region under these conditions. However, inasmuch as the pressure forces at the wall always aid such flow reversals in adverse pressure gradients and always tend to halt them in favorable pressure gradients it is to be expected that such flow reversals will be far less common and less persistent in negative pressure gradients. This point was checked by repeating the tests with very high disturbances using rods and oscillating the hypodermic needle in a region upstream of the throat of the diffuser where the pressure gradient is strongly favorable and the boundary layer is laminar. When this was done, small flow reversals were found in the wall layers of the negative pressure gradient regime. However, they are harder to produce, and they persist for far shorter periods of time. This is interpreted to mean that the continual effect of the favorable pressure gradient is such that while flow reversals can occur near the wall in such a regime, they are far less likely, and they require larger pressure disturbances. Thus in the initial tests which were used to check the technique of dye injection no reversals were found in favorable pressure gradient regimes because the test was not run a great number of times and the disturbance level was only moderate in the free stream.

In order to check the effect of free stream fluctuations further, a set of tests was then also made using a very smooth inlet flow. This was achieved by use of an alternate water supply elimination of all fixed dye injectors, and a careful cleaning and rebuilding of the inlet damping screens. The upstream flow conditions were then very smooth and free from random disturbances; a dye trace injected far ahead of the inlet would in general remain intact in the free stream flow through most or all of the diffuser. At this level of fluctuations in the free stream, transitory stall was much less noticeable and occurred only at the higher angles. That is, for an L/W_1 of 8 no transitory stalls were visible at all for angles less than 5^o, and even small stalls at the wall were hard to find at angles in the range of 10-12^o. The entire character of the flow in the wall layers appeared much evener and more uniformly oriented downstream. Small stalls were still found in the region just downstream from the throat, but these stalls did not fluctuate nearly as much as before and were larger in size. That is, instead of four or five stalls near the throat which moved in location, one or two larger stalls very near the wall appeared and were relatively stable in regard to location. The same thing was observed in stalls observed farther downstream. That is, the stalls that were found appeared as larger flat areas of stall very near the wall and while they were still transitory in nature, the motions were far slower than at the higher disturbance levels.

It is also worthy of note, that in the wall layers where no transitory stall could be observed a different type of self-energizing of the wall layers appeared to occur. This mechanism consisted of the production of vortices, apparently originating in the wall layers, with an axis of rotation parallel to the direction of flow. Such vortices appeared in various locations and had various sizes. They were not fixed in space but appeared first in one location and then in another. After starting near the wall, they move outward as they progress downstream. The origin of these vortices is unknown. It is clear, however, that they will act in a manner similar to vortices introduced artificially by use of vortex generators in energizing the wall layers. Further study of this phenomenon is needed.

It is also instructive at this point to consider the effect of a longitudinal perturbation in the inlet velocity profile of the wall layers. In particular, a small variation of velocity of two adjacent particles in the boundary layer each at the same distance from the wall is of interest. It is clear that such a perturbation could occur as a result of either a fluctuation from the free stream of the sort just discussed or a small irregularity in the upstream history of the boundary layer. For the sake of clarity let us assume that the irregularity in velocity is in the form of a step of size ε. Thus at a given section on the wall the velocity of a particle may be u, while a short distance away, parallel to the wall, another particle has a velocity $u + \varepsilon$. Let us examine what will happen if this flow traverses a positive pressure gradient. The convective deceleration required for a given pressure rise again can be found from Euler's equation as a first approximation. If the particle with initial velocity u is denoted by subscript "a" and that with initial velocity $u + \varepsilon$ by subscript "b", then since each particle traverses the same pressure rise, we can write:

$$dp/\rho = -u\,du_a = -(u + \varepsilon)du_b$$

or:

$$du_a/du_b = (u + \varepsilon)/u \qquad (6)$$

Thus the deceleration of the two particles is different. By letting ε be either plus or minus it is readily seen that faster moving layers are decelerated less than the average and slower moving layers are decelerated more. Consequently, an initial unevenness in velocity profile is magnified by the non-linear interaction of pressure and momentum forces. Furthermore, if the two particles just considered are now made to traverse a still further adverse pressure gradient the difference in velocity will be further magnified at a more rapid rate, ϵ will be larger both in absolute value and in relation to the decreased value of u. Thus the process tends to be unstable. Such an action could account for local three-dimensional stall inception even in a flow which was relatively free from fluctuations in the free stream. This analysis also involves far less approximation than the previous one. Equation (6) neglects the effect of viscosity, but is otherwise exact for a steady motion under the conditions assumed.

The descriptions of the small transitory stall regime just given are partly historical due to the need to establish that such a regime actually exists before discussing the nature of the flow. Since the number of different observations are considerable, it may be desirable to recapitulate the conclusions now available about the mechanisms in this regime of flow, in order to make clear what is known, what is surmised, and what still remains unknown.

The experiments show that small transitory stall definitely does exist. When it exists, it consists of small isolated spots of stall that originate in the wall layers, move upstream a short distance then leave the wall, and finally are carried back downstream rapidly by the momentum forces of the mainstream. The minimum divergence angle at which small transitory stalls can first be observed, the size of the stalls, and the frequency of their occurrence all are strongly dependent on the amount and probably the type of fluctuations present in the free stream. Increase in free stream fluctuations tends to decrease the angle of first occurrence of transitory stall; large enough fluctuations can even decrease the angle to negative values. Such increases in fluctuations also increase both the size and frequency of occurrence of spots of stall. Quantitative studies of the effect of the type and size of free stream fluctuations need to be carried out. For a given inlet condition, that is for all upstream conditions held fixed including the free stream fluctuations, the amount of small transitory stall increases with increased adverse pressure gradient, with increased thickness of boundary layer, and probably also with increased length of application of the adverse pressure gradient.

Small transitory stall is not a different phenomenon from large transitory stall previously reported by Moore and Kline (1) and further described in the next section. As pressure gradient is increased the size of the transitory stalls grow so that the two regimes blend continuously into each other, and the distinction is one of size rather than kind. The distinction is useful not because the two regimes blend together in the middle, but rather because their behavior is different at the two extremes of points 1 and 4 of Fig. 4.

LARGE TRANSITORY STALL REGIME

As already noted, this is the region between curves bb and cc on Fig. 2 and between points 2 and 3 on Fig. 4. In this region large transitory stalls move up and down along the walls. The stalls are frequently larger than the thickness of the boundary layer and cover an entire corner of the passage or a major fraction of one wall. As these large transitory stalls become more numerous, they begin to overlap, and local areas of fixed, that is fully-developed, stalls may occur simultaneously with the large transitory stalls. A study of the description of the flow found by tuft observations pictured in Fig. 3 is revealing in this connection. It clearly shows that a succession of states occurs in which all sorts of various combinations of stall patterns occur. For given inlet conditions, the amount of transitory stall apparently increases monotonically with angle from point 1 to near point 4 of Fig. 4.

At point 4 of Fig. 4 however, it is clear that a change in the flow mechanism must occur from the shape of the curve. Both Fig. 2 and Fig. 4, from the observations of references 1 and 2 respectively, show that this change is to a fully-developed asymmetric two-dimensional stall. However, there are several questions in regard to this change. First of all, "Why does the stall suddenly alter from a three-dimensional transient or transitory type to a relatively steady two-dimensional or fully-developed type?"

This question has been studied, again by water table observations, in some detail: A diffuser flow was set up with a divergence angle slightly greater than that of point 4 of Fig. 4 and with an L/W_1 of 8. The large fully-developed stall was then eliminated from the diffuser by insertion of a cluster of vanes in the manner described in reference 1. After the flow was again apparently unstalled, (that is, all the flow appeared moving downstream when viewed with conventional dye techniques) the vane cluster was given an inclination away from the centerline in order to establish a clear preference of the subsequent fully-developed stall for one wall. The vane cluster was left in this cocked position for about thirty seconds and then removed. Dye was injected over the whole area along the wall on which the fully-developed stall was developing, and the flow was observed to see where and how the fully-developed stall began. This test was repeated about two-dozen times. The first few trials were sufficient to establish that the fully-developed stall began as a fixed spot of stall in the corner just downstream of the throat, that is, in the location where the boundary layer is thickest for the cross-section of maximum adverse pressure gradient. This fixed stall then grew, apparently by simple accretion of fluid from behind, until it extended down the entire corner of the passage. At this point, the entire mainflow at the surface and to down to about half the water depth of 4" was still proceeding downstream on the same wall. Next the stalled area grew upward until the entire wall was stalled thus finally giving the fully-developed stall previously described for this value of divergence angle and L/W_1. The growth action was not fast; a total time of the order of several minutes was required on the average for the stall to build up to the final pattern after the vane cluster was removed.

Initially it had been felt that vorticity might play a large role in the establishment of a fully-developed stall since the transitory stalls create small vortices, and the fully-developed stall appears as a large recirculating region which contains considerable two-dimensional vorticity. However, tests in which vorticity was artificially introduced by a spinning cylinder in various ways completely failed to confirm this idea. In fact, no strong vorticity components have been observed in the growth pattern described above although vorticity does appear to play an important role in the shedding of individual spots of stall from the wall and in the flow along the edge of the slip stream between the main flow and a fully-developed stall region. The action of vorticity in spot stall shedding is not yet understood, and is still under study.

It is clear from the description given above, that the fully-developed stall builds up gradually going first from transitory stall to a three-dimensional type of fully-developed stall and finally to a two-dimensional fully-developed stall. Furthermore, it is clear from the way in which the stall grows larger and larger that a stability problem is indeed involved. The fact that a stability consideration must be introduced is also evident from the curve of Fig. 4. At point 4 on Fig. 4 a very small increase in angle brings about a very large change in the entire pattern of the flow; this in itself is clear evidence that some stability consideration should be introduced. Since it now appears that for fixed inlet conditions and fixed L/W_1 the amount of stalled area tends to increase with increasing divergence angle and since the fully-developed stall is created simply by growth, it would appear logical to attempt to formulate the problem in terms of a balance between the rate at which stalled fluid is produced over a given section of wall and the steady rate at which it can be swept away. As soon as this concept is formulated, continuity considerations can be employed to explain many observed phenomena.

It must be remembered that the production of stalled fluid in the transitory stall regime is not a steady phenomenon; but is instead a time dependent phenomenon which produces spots of stall in an apparently random fashion over both time and space. However, by taking a time average over a long enough period, certain properties independent of the individual fluctuations can be determined. Consider a given segment of wall as shown in the control surface of Fig. 11. For such a control surface, it is evident that the smaller the long term average rate at which stalled fluid is produced inside the control surface compared to the possible rate at which it can be swept away, the smaller the percentage of time a spot of stall will be present inside the control surface, and the smaller the percentage of wall area it will cover on the average. Thus at very low adverse pressure gradients only isolated small spots of stall of short average duration are observed. However, as the adverse pressure gradient is increased, more stalled fluid is produced, and the average area of wall covered by stall over a period of time increases. If this process is continued, a point eventually will be reached at which the rate of production of stalled fluid will exceed the rate at which it can be swept away on the average. Thus stalled fluid will gradually accumulate, and a larger and larger area of stall will build up. This is entirely in agreement with the observed facts in the diffuser studies reported above and also with the movies of flow over airfoils taken many years ago under Prandtl's direction at Göttingen. In both instances, a backflow does not simply set in along a line as envisaged in classical boundary layer theory, but rather a small area of stall first appears. The stall then continues to grow until it reaches its final steady-state size. In addition, both studies show that the final steady state stall point is often not the same as the point from which the growth commences.

As the following discussion will illustrate, this concept of the balance between the rate of production of stalled fluid and the rate at which it can be swept away by the available momentum forces of the mainstream near the wall is apparently central to the understanding of all stall mechanisms. It is therefore pertinent at this point to discuss the nature of the forces by which stalled fluid is produced and by which it is carried away downstream.

In a viscous flow with an adverse pressure gradient there are two forces that tend to produce stalled fluid; these are wall shear and steady pressure forces. There is only one source of forces available for removing stalled fluid, however: this is the momentum or inertia forces of the through flow. However, the momentum forces may act in either of two ways. Transient pressures can be created by local deceleration of the flow, and drag forces, laminar and turbulent may be exerted laterally by the high speed fluid.

Consider a single transitory spot of stall in the small transitory stall regime. If it is an isolated spot of stall on a wall, the forces on it are as shown in Fig. 12a. Since the spots of stall on the wall move relatively slowly upstream, the wall shear will be small. Since the momentum forces near the wall are very low, the primary forces on the spot of stall are then the drag forces on its outer edge which are caused by the downstream flow and the pressure forces pushing it upstream. As soon as such a spot leaves the wall, the drag forces due to the surrounding layers of fluid increase rapidly inasmuch as both the velocity gradient and the area of application increase. The momentum forces due to impact on the upstream end of the spot also increase very rapidly due to the increased oncoming velocity farther from the wall. Since the velocity of the spot of stall is very low, to a first approximation it can be viewed as an obstacle in so far as the main flow is concerned. Thus the pressure at nose of the spot can be approximated by stagnation pressure locally and is proportional to the square of the local velocity as shown in Fig. 12b. Thus it is not surprising that the isolated spots disappear very rapidly once they leave the wall.

On the other hand, if the spots of stall become relatively large, the balance of forces is altered by the fact that the surface area available for drag by the mainstream is proportionately reduced as shown in Fig. 12c. Thus the relative effect of both the shear forces tending to sweep the spot downstream and the rate at which stalled fluid is captured from the spot by entrainment at its boundaries are reduced. Hence the spot would tend to stay in the diffuser longer and to decrease in size more slowly. Another view of the same thing is seen by noting that if only isolated spots of constant size occur then the drag area of each spot is unaltered no matter how many spots appear. In fact, the existence of spots of stall, by virtue of continuity, requires the remainder of the flow to move downstream faster and hence the sweeping-out rate of stalled fluid is somewhat augmented by more spots of stall. However, if so many spots of stall occur that they begin to coalesce, then the effect of the drag forces of the mainstream is reduced. This can clearly be seen by comparing the two types of flow geometries shown in Figs. 12d and 12e. Thus as the spots begin to coalesce, a destabilizing type of action is to be expected. This seems to be in agreement with the flow pictures shown in Fig. 4. That is, the onset of fully-developed stall occurs suddenly at about the point where the transitory stalls are becoming large and begin to overlap and interact.

The following further argument concerning the unstable nature of the transition from transitory to fully-developed stall can also be given. It is known from many observations that the fully-developed stall may start on either wall, and that some apparently trivial difference initially decides the choice of wall. However, in the two-dimensional diffuser geometry, with steady, thin inlet boundary layers no case has ever been observed in which the fully-developed stall flopped from one wall to the other. It can be forced to change walls by introducing a very large disturbance such as blocking the flow down one wall entirely for a period of time, but in the absence of such very large disturbances it appears stable and relatively steady. Consequently, the growth of the fully-developed stall region seems to stabilize the flow. This can be rationalized in terms of the force pictures of Fig. 12d, 12e and 12f. In the case of a fully-developed two-dimensional stall as shown in Fig. 12f, not only is the proportionate drag forces reduced as compared with those on an isolated spot shown in Figs. 12c and 12d, but also the pressure at the leading edge of the stall region is lower than the stagnation pressure which would be exerted on the nose of an isolated spot. In addition, the high speed layers of fluid have now been pushed away from the wall, and can act to remove stalled fluid only along a two-dimensional interface relatively far from the wall. Thus, for all these reasons, as a spot of stall becomes fixed and starts to grow, the rate at which stalled fluid can be swept away from the location in question is reduced, and the spot continues to enlarge itself due to the continued excess of backflow from downstream until the entire flow is pushed away from the wall. The question then arises, "How is a dynamic balance restored, in the fully-developed stall region?" This question is discussed in the next section.

The foregoing arguments on stall mechanisms are not rigorous in form. It will be clear to the reader that additional work, both empirical and theoretical, is needed. However, the hypothesis about the nature of the breakdown of the flow to fully-developed stall seems to fit the known facts very well. What is more important, the concepts developed to provide this explanation can be used to explain many more known facts, and have also already been utilized to predict, at least qualitatively, new results which are verified by experiment. These matters are covered in the following sections. It therefore seems clear that further effort to investigate the mechanisms described is clearly in order, and it is believed that the basic concept of the balance between the rate at which stalled fluid is produced and the rate at which it can be swept downstream will prove fruitful in this work.

Before leaving the matter of breakdown to fully-developed stall, two further comments are in order. In his brilliant paper on the flow of turbulent boundary layers in an adverse pressure gradient Clauser (6) found one result which he was totally unable to explain. This is the matter of downstream instability. Clauser found that as the profiles of the turbulent boundary layer were altered from the type found on a flat plate flows towards those found in nearly stalled boundary layers, at first the tolerable adverse pressure gradient increased, but then it began to decrease again. If the mechanisms described above are accepted as correct, then an explanation of Clauser's observations can be given. Since Clauser's measured profiles represent mean velocities, no indication of the amount of stall produced is given. However, if it is assumed as stated above that the amount of stall produced increases with increased adverse pressure gradient and with increased length of application of adverse pressure gradient throughout the transitory stall regime then it is clear that the destabilizing action due to coalescence of spots of stall might well be the explanation of the instability observed by Clauser.

In view of the discussion given in the section on small transitory stall on the profound effect of free stream fluctuations on small transitory stall, it is also desirable to comment explicitly on the effect of fluctuations on large transitory stall. The tests thus far performed suggest that the effect of fluctuations on large transitory stall is far less pronounced than the effect on small transitory stall. It is noted that line a-a in Fig. 2 is not altered greatly by large changes in free stream fluctuation level. This fact was observed by Moore and Kline (1) and verified by Cochran and Kline (2). There is still some question regarding whether a condition can be created by use of an exceptionally disturbance-free, uniform upstream flow in which stall will actually begin as a two-dimensional phenomenon, that is, in which no transitory stall will appear at all but the flow will alter suddenly from an entirely unstalled to a fully-developed stall condition. This question cannot be answered definitely since there is no limit to the amount of precautions that can be taken to make a smoother and more uniform inlet condition. However, it can be said that for the smoothest inlet flows thus far achieved such a condition has never occurred. In other words, the first stall observed in every case thus far, has been transitory; fully-developed stall then occurs only after a further increase in divergence angle or L/W_1, apparently as the result of growth of one or more spots of stall. It would therefore seem likely that a direct transition from an entirely unstalled condition to fully-developed stall is not to be expected except possibly under the most carefully controlled laboratory conditions or for laminar boundary layers at very low levels of free stream turbulence. Under normal conditions for virtually all internal flows and probably at least many external flows, it is the author's belief that careful observation will show that stall inception occurs as a transient three-dimensional phenomenon.+

FULLY-DEVELOPED STALL

Steady, Asymmetric, Fully-Developed, Two-Dimensional Stall

In the previous section the notion was advanced that fully-developed stall was the result of the unstable growth of a spot of stall which resulted from the fact that the time average rate of production of stall over a given area of wall exceeded the average ability of the locally available momentum forces of the mainstream to remove the stalled fluid. It was also shown that the alteration in forces on the stall spot were such that once such an action commenced it could be expected to continue until a large region of stall or a wake area developed. This was seen to be in accord with the observed facts for both airfoils and diffusers.

A question was then propounded regarding how a dynamic balance was restored. An explanation for this question can easily be given in terms of the balance between the rate at which stalled fluid is produced and the rate at which it is carried downstream. For any steady state flow configuration to exist, even periodically, these two rates must be equal. The term stalled fluid, as already noted, is used in this discussion to denote any fluid that is moving against the main direction of flow. Hence as can be seen by examining Fig. 2 or Fig. 12f, for a fully-developed stall, the stalled fluid includes the entire back flow up the stalled wall.

Consideration is next given to what creates the back flow rate of stalled fluid and what determines how fast it is removed. The rate of production of stalled fluid along

+This, of course, refers to stall on a faired wall. Obviously, a two-dimensional stall can be generated by a sharp corner of large angle.

a given section of wall is believed to be primarily a function of two things: (i) the value of adverse pressure gradient, (ii) the amount of fluid near the wall which is deficient in total pressure. What is meant by "deficient in total pressure" is defined and analysed in detail in the section on losses below.

In a fully-developed stall the amount of stall fluid swept away downstream is controlled by the rate of mixing of the stalled fluid with the mainstream. This in turn is influenced by the nature of the boundary layer (or slipstream) in which this mixing occurs and by the geometry of the stalled region.

With these qualitative concepts in mind it is easy to rationalize the behavior of the steady fully-developed stall region found in diffusers. Referring to Fig. 13a, it can be seen that the pressure in the exit plane of the diffuser is controlled primarily by the size of the stalled region. This is proven as follows.

Since the mainstream flows along one wall with little friction inside the stream, Euler's equation can be applied along a streamline at the center of the mainstream. This yields:

$$p_1 + \rho_1 \frac{V_1^2}{2} = p_2 + \rho_2 \frac{V_1^2}{2} \tag{7}$$

Also, for a steady configuration continuity requires $w_b = w_e$. Therefore: $W_1 V_1 = jV_2$;

$$\text{or} \quad \frac{V_2}{V_1} = \frac{W_1}{j} \tag{8}$$

Combining equations (7) and (8) gives:

$$p_2 - p_1 = \rho \frac{V_1^2}{2} \left\{ 1 - \left(\frac{V_2}{V_1}\right)^2 \right\} = \rho \frac{V_1^2}{2} \left\{ 1 - \left(\frac{W_1}{j}\right)^2 \right\}$$

or

$$C_{PR} = \frac{p_2 - p_1}{\frac{1}{2} \rho V_1^2} = 1 - \left(\frac{W_1}{j}\right)^2 \tag{9}$$

and since $W_2 = h + j$ substituting into (9) yields:

$$C_{PR} = \frac{p_2 - p_1}{\frac{1}{2}\rho\ V_1^{\ 2}} = 1 - \left(\frac{W_1}{W_2 - h}\right)^2 \qquad (10)$$

Thus the larger the value of h, the smaller the value of pressure recovery.

Since the flow in the fully-developed stall region is known to be steady, it follows from the arguments above based on continuity that the rate of backflow up the stalled wall, w_b, must equal the rate at which fluid is removed by the entrainment in the mainstream, w_e. If it is noted that the entrainment flow is relatively insensitive to the pressure at section 2, and also that the backflow in this case depends primarily on the pressure at section 2, then the explanation of the stability of this type of flow pattern is easily given.

If the pressure in the exit plane increases momentarily, the backflow will increase, but the entrainment flow will not be materially altered. Hence, $w_b > w_e$, and by continuity the size of the stalled region must increase with time. This will in turn increase the dimension h and reduce the pressure at section 2 thus tending to reduce w_b and restore a dynamic balance. A similar argument applies when the pressure is reduced at section 2. In this case $w_b < w_e$; h decreases, and the pressure at section 2 rises tending to restore the dynamic balance.

If the foregoing argument is correct, then the rate of backflow up the stalled wall is critical in the performance of the diffuser in this zone. And any change in geometry that tends to alter the backflow rate of stalled fluid would influence the pressure recovery very markedly. This presents a simple means for checking the concept developed. The following experiment was performed in the water table. A fully-developed stall was set up. The dimension h was noted. Then a strip of galvanized steel was inserted into the exit plane so that it blocked the flow across approximately twenty-five percent of the dimension h near the stalled wall (see Fig. 13b). As soon as this was done, the size of the stall began to decrease. After a few minutes it had decreased forty to fifty percent; it then remained fixed in size. As soon as the blocking vane was removed from the exit plane, the stall zone returned to its original size. This experiment was then repeated several times. The results were the same each time.

It is believed that this verifies the concept developed concerning the nature of the flow in a fully-developed stall region. The experiment indicates that in this regime, control of the pressure recovery can be achieved at least to some extent by reduction in backflow rate as well as by increase of the rate at which the stalled flow is removed. This again reinforces the idea that it is the ratio w_b/w_e rather than the individual values that is important in governing the behavior of a stall.

This experiment also shows that the flow can be strongly affected by downstream geometry, that is by the geometry downstream of the point of separation, in a fully-developed stall regime. This fact probably accounts for all or a large part of several known results. First of all, the scatter in the diffuser performance data in the fully-developed stall regime is enormous. In this region recoveries varying from 5 to 50% have been recorded for what appeared to be similar conditions, that is, for comparable values of divergence angle and length to width ratio. It is quite likely that a large portion of this variation is due to variations in downstream geometry that are usually not considered in diffuser design. These variations in downstream geometry can affect the stability of the overall flow pattern by altering the ratio w_b/w_e in the fully-developed stall region, and as shown by the example above the results may be in the opposite direction from that which would be predicted for an unstalled flow.

It should be noted that these effects of downstream geometry are probably only important in the fully-developed stall region. They will be much smaller or non-existent in the transitory stall regime, for unstalled flows, and in well vaned diffusers. It is also apparent that in some cases downstream geometry variations on the unstalled wall may be less significant than on the stalled wall in so far as improvement of performance in the fully-developed stall regime is concerned. It is also quite clear that further study of this type of behavior is needed in order to evaluate these concepts quantitatively and to obtain further details.

Several other important conclusions can be drawn from the ideas just given. First, while the concept of the balance between backflow rate of stalled fluid and downstream flow by entrainment is basic to both zones of stall described, in several other regards the concepts needed to explain the behavior of the two regimes are very different. In particular, in the case of the asymmetric, steady, fully-developed, stall consideration of the stability of the entire flow pattern and its dependence on the interaction between the mainstream and the stalled region must be introduced. This is a concept that transcends classical boundary layer theory; it will be found that similar considerations are needed in the explanation of the other zones of fully-developed stall. This discussion also illustrates again the futility of discussing simply "stalled" and "unstalled" flow. Since the concepts required to explain behavior in the different zones of stall are in part different, it is quite clear that the type of stall present must be known if any rational picture of the flow behavior is to be formed.

Perhaps of even more practical importance than the conclusions just stated is the fact that alteration of the backflow rate of stalled fluid can alter the behavior of the stall. This concept will be used to advantage again below not only to rationalize known behavior but also to predict means for improving real flows. From the general point of view, if the central concept governing the behavior of stall is the balance between the rate of production of stalled fluid and the rate at which it is swept downstream, then it is clear that this balance can be improved in either of two ways: (i) increase the forces available near the wall for sweeping stalled fluid downstream, (ii) decrease the backflow rate along the wall. In most of the means for control of boundary layers and wakes proposed to date explicit use is made only of item (i). Item (ii) has frequently been used implicitly, but there are some situations in which it should be possible to obtain considerable improvement in the flow by explicit use of item (ii). This will be discussed in more detail in the section on Control of Boundary Layers and Wakes.

Jet Flow Regime

In terms of the classification of stalls given above the jet flow is also a steady fully-developed stall. It has two fully-developed stall regions instead of one, however, as shown in Fig. 2. As previously noted, there is again a stability problem associated with the transition from the two-dimensional asymmetric stall to the jet flow. Some additional facts on this stability problem are now given.

If the slow build up of the asymmetric stall is observed in the water table under symmetric conditions, that is without the purposeful inclination of the initial flow described in the tests above, then the amount of transitory stall actually increases on both walls more or less symmetrically as angle is increased. At the point where the breakdown to fully-developed stall occurs, very large transitory stalls with some areas of fixed stall appear on both walls, and this may continue for several minutes, or in some cases, even longer if the upstream flow is particularly symmetric. Under these conditions, the mainflow moves primarily down the center of the channel. Then some relatively large random disturbance or some residual initial effect starts the mainflow moving from the center toward one side. As shown in Fig. 14, this creates a larger pressure on one side of the mainflow and a smaller one on the other due to the curvature of the streamlines. Hence the backflow up the wall toward which the jet moves is decreased, and the backflow on the other wall is increased. The stall on the wall toward which the jet moves therefore tends to decrease in size, and the stall on the other wall increases in size. This pushes the mainflow still further from the center; a larger streamline curvature occurs, and the process is continued until one wall has a fully-developed stall and the mainstream runs down one wall. Thus a jet in the center of two walls tends to be unstable under these conditions. This explains why an asymmetric flow is found over a wide range of angles.

If the angle is continuously increased, the wall moves farther and farther from the jet, and ultimately a point will be reached where a small change in jet shape will no longer materially affect the pressure distribution along the downstream wall. Then the interaction just described will not occur, and the jet regime will remain stable.

The explanation given above regarding the establishment of the asymmetric fully-developed two-dimensional stall also can be used to explain the overlap region found at the transition to and from jet flow shown by lines cc and dd on Fig. 2. When angle was increased, the transition was found at line cc; when angle was decreased, the transition was found at line dd. As angle was increased the transition at line cc was slow; the mainstream fought its way out into the center gradually often requiring more than five minutes to effect the entire transition. On the other hand, when angle was decreased the transition at line dd was relatively rapid which illustrates the unstable nature of the jet flow when the confining walls are close to the jet.

The nature of this instability has also been investigated in an experiment described to the author by H. Emmons. This experiment consists of running a small tube from the stalled wall to the unstalled wall as shown in Fig. 15. This tube equalizes the pressures on the two walls and thus eliminates the instability caused by the pressure difference across the curved streamlines of the jet. Thus jet flow is stabilized to much lower angles of divergence.

UNSTEADY FULLY DEVELOPED STALL

Let us return now to the question, "Will a fully-developed stall remain steady, or will it periodically shed stalled fluid?" The answer to this question is readily found from the concepts already developed. Consider a stall such as that shown in Fig. 16a. If the backflow rate w_b is greater than the entrainment rate w_e, then the size of the stall must grow. If this process is long continued, a bulge of stalled fluid will ultimately begin to block the mainflow, as shown in Fig. 16b. This bulge acts as an obstacle to the flow. It will consequently have a low pressure region behind it, and a high pressure region in front of it. Thus a point will soon be reached when the bulge will be ripped off by the momentum forces of the flow and proceed downstream as a vortex. The fact that this must be so is easily seen since if it were not true then the stall would soon block the entire flow. In other words, the growth of the stall region brings into play the same increased forces that act to remove isolated spots of stall once they have left the wall, that is, the momentum forces of the mainstream acting through both impact and shear.

This conception of the unsteady, wake shedding, stall should be a useful one in that it suggests means for reducing or even eliminating the wake shedding process. For example, in the case of the corner two possible schemes are shown in Figs. 16c and 16d. These configurations have not been tried, and are undoubtedly not optimum ones. The scheme of Fig. 16d is somewhat more complicated, but it offers greater control over the steadiness of the stalled region because the ratio w_b/w_e can actually be controlled. Such schemes may be of practical utility in certain situations since it is well known that the loss due to a stall region is not great, nor are large fluctuations caused in the flow if the stall remains fixed or steady. It is only when the stall or wake sheds large hunks of fluid as vortices, that really great losses and large flow fluctuations occur.

The following simple experiment related to wakes has been performed in this connection. A cylinder of 0.500" diameter was inserted normal to the flow in the water table. The Reynolds number was approximately 3000 so that the flow regime was well above the upper critical value found by Roshko (9). As usual a wake of a highly fluctuating nature was then found upon injection of dye upstream from the cylinder. This wake was about three cylinder diameters in width at a station four diameters downstream from the back of the cylinder. After making this observation a cylinder of 0.041" diameter was inserted in the wake in such a way that it tended to impede the backflow into the wake region. The most effective point was found to be along the downstream centerline slightly forward of the point where the free streamlines coming from the stall points appear to rejoin. With the small cylinder in this location, a striking reduction in both the size of the wake and the flow fluctuations was observed. The wake was reduced to approximately one-third to one-half of its original size (the exact amount is hard to determine because of inability to define the wake region accurately with dye). The oscillations were not eliminated entirely, but the size was much reduced and the frequency also appeared lower. Insertion of a rod of 0.100" diameter in the same location as the 0.041" rod was even more effective in reducing the size of the wake, and actually appeared to eliminate most of the fluctuations; shedding would only occur now and then, apparently when a strong perturbation came from upstream. Insertion of a flat plate 1/16th inch thick and one inch long on the downstream centerline of the cylinder starting from the trailing edge was even more effective. The reason for this is believed to be that the plate not only reduces the backflow up the centerline, but also prevents large disturbances from upstream starting an oscillation of the stalled fluid from side to side, and thus placing it where it will be washed downstream by the momentum forces of the mainstream. This last result is in agreement with similar results reported by Roshko (9). These results also suggest that useful information may be found concerning the problems of the fluid induced oscillation of cylinders such as smoke stacks and periscopes by considerations of this kind. It has already been noted by Ozker and Smith (10) that several smokestacks in a row give very different results than single stacks. Price and Thompson (11) also found that the oscillations were markedly altered when large amplitudes of the cylinder were admitted. In terms of the concepts just discussed this is to be expected inasmuch as a large lateral movement of the cylinder would expose previously developed stalled fluid directly to the momentum forces of the mainstream. It would thus tend to cause a large alteration in the ratio of production rate to removal rate of stalled fluid.

Similar arguments can readily be applied to the case of the spinning cylinder normal to the flow as compared to the stationary cylinder. Examination of the movies prepared years ago under Prandtl's direction for these two cases shows that the wake produced by a spinning cylinder is much smaller and steadier than that for the stationary cylinder. In terms of the present concepts this would also be expected from the fact that the stagnation point occurs at the rear of the cylinder in the stationary case and on the side for the rotating case shown. Thus in the rotating case the stalled fluid is exposed directly to the mainstream flow, and is swept away downstream nearly as fast as it is produced. Consequently, the growth and decay of the wake region does not occur; the flow is much steadier and the wake is smaller.

CRITERION FOR EXISTENCE OF VARIOUS TYPES OF STALL

The foregoing discussion and description of experiments are not adequate to answer all of the important questions relating to the many problems discussed. Much work remains to be done. However, it is believed that they are sufficient to show that the concept of the dynamic balance between the rate of backflow and the rate of downstream flow of stalled fluid is central to the rational understanding of the behavior of stalls. Furthermore, in terms of the foregoing discussion it is now possible to give a more definite basis for the classification of stalls than that given above. The discussion shows that the real basis for the classification suggested is as follows. If w_b is defined as the time average rate of backflow over a given section of wall area, and w_e is defined as the time average rate at which this stalled fluid can be swept downstream <u>steadily</u> by the available momentum forces of the mainstream flow, then the following criterion applies:

$w_b/w_e \ll 1$	Small Transitory Stall
$w_b/w_e < 1$	Large Transitory Stall
$w_b/w_e = 1$	Steady Fully-Developed Stall
$w_b/w_e > 1$	Unsteady (shedding) Fully-Developed Stall

RATIONALIZATION OF OTHER KNOWN RESULTS

VANED DIFFUSERS

Since the initial purpose of the present work was to explain the excellent results found with vaned diffusers, it is appropriate at this point to apply the concepts developed to this problem.

The problem as stated above is, "How can a vane that does not alter the pressure distribution of the potential flow have any large effect on the flow conditions?" The answer to this question can now be given readily. As shown above, a fully-developed stall apparently does not ordinarily begin along a line in the way described by classical boundary layer theory but instead grows from the unstable enlargement of a spot of stall. Furthermore, this unstable growth apparently arises from the fact that the spot of stall in growing pushes the mainstream away from the wall and thus decreases the momentum forces of the mainstream available at the given wall section. This in turn reduces the rate at which stalled fluid is swept downstream alomg that portion of wall. If a vane is now inserted in the flow in such a way that it tends to prevent the mainstream from being diverted away from the wall, apparently, this unstable action can be prevented. A sketch of the type of vane configurations employed is shown in Fig. 17a (for complete details see Reference 2). It is clear that these vanes will indeed tend to prevent the mainstream from being forced away from one wall. The action of a single vane is pictured in more detail in Fig. 17b. When the flow is steady the vane does very little. However, if a large transitory stall began to destabilize the flow, the vane would then have an angle of attack relative to the mainflow and would create a pressure distribution of the sort shown in Fig. 17b. This pressure distribution is of a restoring nature; that is, it tends to force the mainflow back toward the wall on which a fully-developed stall would otherwise begin to grow. Hence the effect of the vanes is of a transient stabilizing nature, and they can and apparently do provide the forces necessary to stabilize the transitory stall regime and prevent development of large areas of stall.

If the foregoing description is true, then the vanes should not be expected to eliminate all stalls, but rather should tend to prevent the formation of relatively large stalls. Thus it would be expected that transitory stall will be found on the vanes and on the side walls. This point has been checked in both the air unit and in the water table, and such is indeed the case. The vanes reduce the size of the stall at any one section, but transitory stalls of various sizes are consistently found both on the vanes and the walls. Apparently, then the vanes hold each passage well down into the transitory stall regime. The resultant overall performance is relatively good, and the flows are relatively smooth.

The foregoing explanation is not in conflict with the overall design type of explanation of vane action given in references 1 and 2. This overall explanation simply states that each passage should be such that it will not stall out according to the pertinent single passage data (in this case the data of reference 1). The more detailed explanation just given shows in addition that the overall flow pattern thus created is stable. Hence under these conditions a good flow is found in each passage, and one does not find one or more of the passages stalled out with the entire flow passing through only some of the passages. However, if the individual passages are such that they would in themselves tend to have a fully-developed stall as sometimes occurs when full length splitters are used, then at least one of the passages will stall out completely. This will reduce the pressure recovery and thus reduce the backflow in the other passages; it may then be possible for them to operate without fully-developed stalls. The argument proceeds similarly to that given for the fully-developed stall region above, and is merely the familiar "flow blockage" concept that has been employed for some time in connection with cascade analysis.

Similarly, if the individual passages operate in the very large transitory stall region for the symmetric flow, then some relatively small perturbation could stall one passage entirely. If this occurred, the resulting flow would be stable in the other passages, and the configuration would remain. The design criteria found by Cochran and Kline (2) show that the optimum performance occurs when each vane passage lies approximately at the line between small transitory stall and large transitory stall. This is in agreement with the conclusions just reached from consideration of the mechanisms.

The foregoing discussion also explains why the vanes operate best when placed along streamlines rather than at an angle of attack as was at first felt to be desirable. Since the action of the vanes is not to energize the boundary layer, but instead is to prevent the unstable growth of stall, it is clear that better performance will be obtained if the vane loss is minimized consistent with the stabilizing action. This occurs when the vanes are located on streamlines of the flow. This same argument explains why the vanes give the best performance when the angle of optimum effectiveness is used for each vane passage. As shown in reference (2), the minimum loss in a diffusing passage with a given pressure recovery occurs when the function $1-\eta/\eta$ is a minimum, and this minimum is coincident with maximum η.

UNVANED DIFFUSERS

There are still a few questions relating to unvaned diffusers that have not been answered in the process of developing the concepts above. One of these is, "How does the stall develop that creates the jet flow from the asymmetric, two-dimensional, fully-developed stall at line cc in Fig. 2?" In terms of classical stall concepts, this question is very difficult if not impossible to answer since the flow now proceeds, apparently unstalled down one wall, and the pressure recovery is very low. Thus no reason can be seen why it should ever stall off the second wall. However, it is shown above that in the fully-developed two-dimensional stall regime, there is some pressure recovery, and also that this recovery is governed primarily by the size of the stall region. Consequently it is clear that some positive pressure gradient exists on both walls. Thus in addition to the fixed stall on one wall, some transitory stall is to be expected on the other wall. Examination of the flow using dye injection onto the apparently unstalled wall shows that this is indeed the case; transitory spots of stall do exist. Thus if the angle is made very large near the wall so that the influence of the far wall on the pressure distribution of the stalled wall is small, then the spots of stall can grow into a fully-developed stall in the way described above. In other words, as the walls are spread apart, their mutual stabilizing influence is diminished, and unstable stall growth can take place on the second wall also.

Another questiom that can now be answered relates to the matter of tailpipes on diffusers. It has been found in many tests that addition of a tailpipe to a diffuser of poor performance tends to increase the recovery. In terms of the concepts developed above the explanation to this effect is evident. The addition of the tailpipe tends to reduce the backflow rate of stalled fluid into an already existing stall area due to the pressure forces, but it does not decrease the rate at which stalled fluid is swept downstream by entrainment. In fact, it may increase this rate by increasing the interface surface on which shear forces can act on a stall. Thus the tailpipe will alter the value of w_b/w_e in such a way that a higher recovery is necessary to restore a steady state condition.

WAKES AND BASE PRESSURE

It is clear from the discussion of the contraction corner and the experiment on the circular cylinder described above that the concepts developed are also of utility in the base pressure and wake problems. The same criteria regarding stall exist, and all data available to the author, including airfoil studies in the water table suggest that the same phenomena found in the diffusing passages also occur in positive pressure gradient external flows with the single exception of the stabilizing action of nearby walls.

When the point was reached in the present work where detailed consideration of wakes and base pressures was to be undertaken, a conversation was held with Dr. D. R. Chapman of the Ames Aeronautical Laboratory of the NACA whose work on base pressure is well known. It was found that Dr. Chapman and his co-workers had already independently reached many of the same conclusions found by the present author for the case of the truly steady, fully-developed stall. In fact, Chapman, Kuehn, and Larson (12) presented a paper at the Ninth International Congress of Applied Mechanics (Brussels 1956) on the detailed analysis of certain cases of this type of stall. This brilliant work explicitly uses the criterion $w_b/w_e = 1$ for a truly steady stall. It also makes numerical calculations of the pressure in the stall region based on Chapman's earlier work (13); these quantitative results are verified by data. This paper therefore gives a complete and remarkably good quantitative verification of the ideas regarding stalls set forth above for the particular case of the truly steady fully-developed stall. Thus it not only provides an important check for the present work which attempts to apply these same concepts to a wider range of stall problems, but it also gives an excellent means for computing quantitative results for at least some cases of the type $w_b/w_e = 1$.

It is worth noting here also that Chapman, Kuehn and Larson starting from the case of the fully-developed, truly steady stall in external flow, arrived at the same parameter as the present author who began by consideration of transitory stall in internal flow. One other similarity of the work of Chapman, Kuehn and Larson (13) to the concepts developed for use in diffusers is discussed in the section on losses.

Since the work of Chapmen, et al goes considerably farther than that undertaken by the present author for the case of the steady fully-developed stall and its connection to the base pressure problem, there is no need for further discussion of the matter here, and it is possible to pass on to other problems.

LOSSES IN DIFFUSING PASSAGES

As noted in the foregoing discussion, the existence of the transitory stall mechanism provides an explanation for the long standing discrepancy in the data between the measured losses and the known values of wall shear and turbulent energy dissipation. The present discussion explores the question: "Why does this loss arise?" and it suggests means for estimating the magnitude of the loss in the small transitory stall regime. The estimates are not carried out in the present work, but will be the subject of later discussion.

As noted above the mechanism of transitory stall can be viewed as a self-induced augmented mixing process that provides the energy needed for the wall layers to move through the adverse pressure gradient. Basically, the answer to the question: "Why is such energy needed?" has been known for a long time It is simply the fact that some fluid is present in the diffuser (or more generally in the adverse pressure gradient zone) that has a deficiency of total pressure due to viscous effects in the diffuser or upstream. However, consideration of the mechanism of transitory stall suggests that a slightly different view than the classical one might be of utility; this is as follows. Any particle of fluid in the diffuser that has a total pressure less than the exit pressure cannot move out the end of the diffuser unless a certain minimum amount of work is done on the particle. This follows from the fact that if the particle is decelerated to zero, it still will not have a pressure as great as the exit pressure of the diffuser. An explicit proof of this fact is now given. We begin with the energy equation for the steady motion of a particle through a control surface in the form:

$$dQ = dh_F + dW_x \tag{11}$$

By definition $h_F = e + pv$, and dW_x is the work done by the particle exclusive of flow work. In the present case we are considering adiabatic flow, and hence dQ is zero. Noting that the only internal energies stored in the particle are the internal thermal energy, e_t and the kinetic energy, $V^2/2$, we can then write Eqn. (11) as:

$$dh_F = de_t + pdv + vdp + VdV = -dW_x \tag{12}$$

where the minus sign simply denotes work input. Furthermore, for one pure substance in the absence of effects of fluid shear, capillarity and external fields, and for the observer who rides on the particle, the First Law, Second Law and State Principle of Thermodynamics require that:

$$Tds = pdv + de_t \tag{13}$$

For an adiabatic process the Second Law of Thermodynamics also dictates that the entropy may not decrease. Consequently, the sum $de_t + pdv$ must be greater than or equal to zero; the greater than applies to any irreversible and the equality to any reversible process. Under the best possible conditions, that is, for a reversible process, a given particle must therefore obey the equation:

$$vdp + VdV = -dW_x \tag{14}$$

If the work input is zero, the maximum possible pressure rise is:

$$p_o - p = \int_V^o - \frac{V}{v}\, dV \tag{15}$$

Eqn. (15) is, of course, nothing but Euler's equation, and could have been used at the outset. However, if the more general form equation (14) is also used, the minimum amount of work required for a given pressure rise can be found. Suppose that the following process is carried out. Let the particle moves downstream reversibly, adiabatically, and without work interchange until its velocity is zero. The pressure rise created is then given by equation (15) and the final pressure achieved is stagnation pressure by definition. However, it was assumed initially that the particle has a stagnation pressure less than the exit pressure of the diffuser. Since the velocity, V, of the particle is now zero, equation (14) dictates that any further pressure rise can only occur if

$$-dW_x > 0. \tag{16}$$

That is a certain minimum work input is required. The value of this minimum work input is the same for any reversible path between the given end states and it may thus be found for the process just discussed by equation (14). It is consequently:

$$-(W_x)_{min.} = \int_{p_o}^{p_{exit}} vdp \tag{17}$$

For an incompressible flow, equation (17) integrates immediately to give

$$-(W_x) = v(p_{exit} - p_o) \tag{18}$$

or more generally, we can write by use of the mean value theorem:

$$-(W_x) = \bar{v}\,(p_{exit} - p_o) \tag{19}$$

where $\bar{v}$ is a suitable average specific volume which will always lie between the extreme values of v involved in the integration. Since v is always greater than zero by definition, no ambiguity of sign can occur in either equation (18) or equation (19). It therefore follows that the requirement of a work input exists only for particles that have a deficiency in total pressure. Furthermore, the minimum work requirement is, at least to a first approximation, proportional to the difference between total pressure and exit pressure. It is also evident that it is impossible for such a work requirement to exist in a favorable pressure gradient zone since by definition if $\frac{\partial p}{\partial x} < 0$ then:

$$p_{exit} < p < p_0 \tag{20}$$

This discussion also yields some information on the effect of inlet boundary layer on diffuser performance. The available data on this point are scanty; however, what data exists, most notably that of H. Peters (14) and Little and Wilbur (15) show that a strong effect exists. To date the tendency has been to correlate the data on the basis of the boundary layer displacement thickness. From the foregoing discussion it would seem that a better parameter could probably be formed by comparing the minimum work requirement given by equation (17) or (18) for the entering low energy fluid to the total entering kinetic energy. The computation of such a parameter requires no greater effort than the computation of displacement thickness of the boundary layer. If a boundary layer thickness measure is to be used, the discussion suggests that the energy deficiency thickness might be most appropriate. However, even this parameter is not a proper measure since equation (18) shows that the work requirement is not only a function of the entering velocity profile but also is strongly dependent on the exit pressure and hence on the geometry of the diffuser. Since the energy deficiency and the displacement thickness are related quantities, it is not surprising that a change in performance has been found when δ^* is varied within a homologous set of data.

Let us now remind ourselves of the various ways in which a given particle can occur in a diffuser with a total pressure less than the exit pressure. These are: (i) it enters with a very small total pressure, (ii) its total pressure inside the diffuser is decreased by wall shear, (iii) its total pressure inside the diffuser is decreased by viscous effects arising from fluctuations in the flow. Data are available from which the deficiency of total pressure created by wall shear can be estimated. Means are also available for estimating the energy dissipation due to normal turbulent decay, as illustrated for example by Kalinske (7). However, until now no account has been taken of the fluid that enters the diffuser from upstream already deficient in total pressure due to the history of the previous boundary layer.

In many cases the largest component of the fluid which is deficient in total pressure will come from the upstream boundary layer. It therefore appears imperative to estimate the effect of this deficiency on performance if rational calculations of diffuser losses are to be carried out. From consideration of continuity, it is clear that for any steady flow, all of the fluid that enters the diffuser with a deficiency of total pressure must ultimately be carried on downstream. If this were not true, then low energy fluid would be continually piling up inside the diffuser and a steady state could not exist. In other words, in the transitory stall regime, by definition, the rate at which stalled fluid is produced is less than the rate at which it can be swept downstream. Hence an estimate of the loss incurred by the action of transitory stall can be found by computing the minimum work required to provide the deficiency of total pressure in the entering fluid, and estimating the irreversibility associated with performing this work in a mixing process. If the dissipation due to wall shear and turbulent decay is added to this transitory stall loss a crude estimate of diffuser loss can be found.

What is perhaps more important than the direct diffuser application just discussed is the more general application of the idea that work must be done on the fluid with a total pressure less than that of the exit pressure in a region of adverse pressure gradient and that this work is done primarily by mixing and shear from the surrounding layers. This idea is an old one, but it is not clear whether the full implications of it have been understood. Since the required work is done primarily by shear, it must inherently involve dissipation of energy and hence a loss in performance. Thus the total loss is not given by the wall shear alone as it is in the case of the favorable or zero pressure gradient flow. The loss apparently usually occurs in mixing between the mainstream and spots of stall. Any forces exerted in such a case will consequently appear as an action by the mainstream fluid with an equal and opposite reaction on the spot of stall. Thus the forces will not appear at the wall unless the spot of stall is moving along a wall. It is likely that it is this added loss that invariably makes the efficiency of diffusing passages lower than those of apparently comparable nozzles despite the higher wall shear coefficients of nozzles.

A closely related idea to that just developed has been recognized very clearly by Chapman, Kuehn and Larson (12) in their paper on the steady fully-developed stall. In fact, one of the critical ideas in that paper is the use of the idea that particles with a deficiency in total pressure cannot escape from a stalled region by their own action. This idea is used to locate the dividing streamline between the steady stall and the mainstream.

The present discussion provides the explicit thermodynamic proof of the correctness of the concept, and shows that in a more general sense it applies to the processes in in any adverse pressure gradient flow.

The foregoing discussion also illustrates, in a different way, the reasons for the behavior predicated, and actually found in the small transitory stall regime. From the discussion of losses, that is energy, just given it is to be expected that for other conditions held constant, each of the following will give increased amounts of transitory stall:

1) Increased adverse pressure gradient.
2) Increased length of application of adverse pressure gradient.
3) Thicker inlet boundary layers.

These conclusions are all in accord with those reached from consideration of the forces on the wall layers.

In addition to these conclusions, the foregoing discussion also suggests, but does not prove, that increased production of stalled fluid will give an increase in total dissipation in the flow. Cochran and Kline (2) have shown that the following relation between head loss and recovery holds for an incompressible one-dimensional diffuser flow:

$$\bar{H}_L = \frac{H_L}{\frac{1}{2}\rho V_1^2} = C_{PR_i} - C_{PR}$$

This function is plotted in Fig. 18 from the data of reference 2. It is seen that it is indeed a monotonically increasing function with increasing divergence angle. This is quite meaningful up to the angle that corresponds to that for optimum recovery, that is between points 0 and 2 in Fig. 4, and in this region it needs no special interpretation. However, for the region beyond optimum recovery, it must be interpreted with caution for the following reason. The loss of head $\bar{H}_L$ is a measure not only of dissipation in the flow but also of the leaving loss. That is it charges the diffuser not only with the head loss due to dissipation but also with the excess of kinetic energy leaving the unit above that which would occur with a one-dimensional exit velocity profile.

For some purposes this is the measure of loss desired, but for other purposes it is not. In particular, if a tailpipe or downstream duct is added to the diffuser some of the leaving loss arising from non-uniformity of velocity profile at the exit of the diffuser will usually be recovered downstream.

It is not yet clear whether a uniform increase in dissipation occurs as angle is increased. It is clear that the actual dissipation will increase uniformly with angle up to point 4 in Fig. 18 where a fully-developed stall first occurs. Thermodynamic considerations suggest that the energy dissipation (exclusive of excess leaving loss) is probably less in the fully-developed stall regime than that found in the large transitory stall regime just to the left of point 4 of Fig. 18, but detailed calculations on this point have not yet been carried out.

COMPRESSOR BLADE UNDERFILING.

One of the mysteries of the compressor and pump industries is the matter of blade underfiling. Empirically, it has been known for a long time that the output head can be increased a modest amount by beveling the exit edge of the blades. This action causes a slight drop in efficiency and is therefore normally only used when the unit does not quite meet specifications of head or pressure ratio. No reasonable explanation of the behavior is known. In terms of the mechanisms discussed above a possible explanation can be given. This explanation has not been checked, and should be considered as a possibility rather than a fact.

If the data on the flow patterns in normal centrifugal impellers taken some years ago by Fischer and Thoma (16) is examined, it clearly shows that the impeller operates at the design condition with each passage in the flow regime that is here called steady, fully-developed stall. Far off design, this configuration apparently becomes unstable and a rotating stall results. However, for the present purposes let us consider the design condition.

Due to the pressure differences created by blade loading, the stall in the impeller is on the suction side of the blade and the throughflow is on the pressure face. We have already seen that changes in downstream geometry, can affect the performance in this zone quite markedly if they are of such a nature that they alter the ratio w_b/w_e. In the case of the impeller flow shown schematically in figure 19, filing of the trailing edge of the blades could alter this ratio. This might occur in the following way. The sketch of the flow pattern in Fig. 19 shows some recirculation or backflow from the pickup region of the diffuser along the suction edge of each blade. In an efficient unit this flow cannot be large compared to the mainflow since it has to be pumped back out again. That is the fully-developed stall in the passage is apparently nearly a closed truly steady one at the design point, and hence most of the flow in the stalled region simply recirculates in a closed loop. However, if any backflow enters from the diffuser pickup due to the pressure forces, then such a flow is impeded by the shear forces exerted on it by the mainflow from the pressure surface of the next passage. If the blade trailing edges are filed, then the distance between the backflow and the outflow along the pressure surface of the next passage is reduced and the effect of the shear forces is increased. This would tend to raise the output head due to the action described in detail in connection with a diffusing passage discharging into a plenum chamber above. That is, the reduction in the ratio w_b/w_e causes a reduction in the size of the stall, an increase in diffusion, and finally a restoration of the balance between w_b and w_e at a higher value of pressure recovery. At the same time, the action of shear between the backflow and the outflow from the next passage is a very strong one, and a reduction in the width of the slipstream will cause an appreciable increase in dissipation with a concomittant decrease in efficiency.

CONTROL OF BOUNDARY LAYERS AND WAKES

A large number of different means for the control of boundary layer flows have been suggested and/or tried during the past fifty years. Most of these schemes have been successful to at least some degree. It is therefore pertinent to see if the mechanisms discussed above check the known results and if they suggest any further possible improvements. All of the methods of boundary layer control previously proposed rest on one of two actions: (i) augmentation of the energy of the fluid near the wall, (ii) reduction of the amount of low energy fluid near the wall by removing it. Both of these concepts are used primarily in a qualitative empirical fashion in practice, and since they are qualitatively totally in agreement what has been predicted from the concepts developed above, no disagreement or conflict whatsoever is apparent. The old idea of "boundary layer health" expressed in terms of velocity profile remains as previously a profoundly utilitarian concept. However, the mechanisms discussed above also suggest certain other possibilities that may be of further use.

In the first place, once it is recognized that more than one type of stall exists and that the mechanisms are in part different for these different types of stall, it is clear that the proper medicine can be more readily prescribed for the proper disease. For example, the results cited above show that increased turbulence in the entering stream increases the angle at which fully-developed stall occurs markedly, but tends to decrease the angle at which small transitory stalls first occur. This is entirely reasonable in terms of the mechanisms of transitory stall and the breakdown of transitory to fully-developed stall discussed above. However, it is equally clear that increased fluctuations in the entering stream may be either helpful or detrimental, and this decision cannot be made unless the nature of flow regime is known.

Another point, and this is perhaps a more important one, is the fact that in the fully-developed stall regions the control of the backflow rate by control of downstream geometry is of great importance. A discussion of this point has already been given in the sections on fully-developed stalls. However, it is worth noting explicitly that the means suggested for control of shedding, and/or recovery in that flow-regime are not derivable from the older concepts based on boundary layer theory alone. In order to reach the conclusions derived in that section it is necessary to recognize the fact that under some circumstances decreasing the backflow rate of stalled fluid may be far easier to accomplish and equally as effective as increasing the rate at which stalled fluid is removed; it is the ratio of the two rates that is critical in determination of the flow regime. It is also necessary to recognize that the interaction between the free stream and the boundary layer is of great importance particularly in the fully-developed stall and wake flows where it apparently determines the stability of the entire flow pattern.

Another idea that is probably explicable in terms of the concept of the ratio of forward flow by entrainment to backflow of stalled fluid is the concept that has been called "pressure filling" in some of the literature. This is a concept that has never been clear, at least to the author. Some comments on this type of action can be made in terms of the mechanisms of stall discussed above. Suppose that a passage is such that it would have a fully-developed stall if it had an open end as shown in figure 20a. Compare this geometry with that shown in figure 20b in which the end of the diffusing passage is filled with a partial obstruction such as a tube sheet of a heat exchanger. Let us then assume that a fully-developed stall existed in the configuration of figure 19b as indicated by the dotted lines. Such a configuration would cause a relatively large pressure drop between points a and b of figure 20b. Since the tubes require at least nearly straight streamlines in the plane of points b and c, the pressure at point c would be approximately the same as the pressure at point b. Thus the pressure at point c would be considerably less than the pressure at point d. As a result, there will be a flow from the region around point c toward the region near point c. This flow is in distinct contrast to the backflow that would occur from downstream toward point d in the absence of the tube sheet as shown in figure 20a. In other words the tubes at the lower end of the sheet act very much the same way as the bleed from the stalled region suggested in connection with the control of stall in corners. The net effect of the tube sheet is thus to reduce the backflow w_b strongly as compared to a similar situation in figure 20a, and hence it is not surprising that frequently the fully-developed stall is entirely eliminated.

Two additional comments on this action are pertinent. In the first place, if the mechanisms described in the results above are applicable in figure 20b, then it is to be expected that transitory stalls will usually occur in the wall layers of the passage even though no fully-developed stall is present. If this is so then the effect of the tube sheet is similar to that of vanes in that it changes the stability pattern of the overall flow configuration. However, it does so by reducing the value of w_b, while the vanes apparently act by preventing a reduction in w_e locally.

In the second place this discussion again emphasizes the importance of downstream geometry in situations which do or might have a fully-developed stall. It also emphasizes the fact that it is necessary to consider the stability of the overall flow pattern in rationalizing the behavior of viscous flow patterns that might involve large areas of stall. This consideration does not replace the classical concepts of the effect of the pressure distribution of the symmetric potential flow on the boundary layer but is supplementary to it. The reasoning is very analogous to that employed to justify the Kutta condition in subsonic airfoil theory. In that case, of the possible potential flow solutions the particular one that satisfies the Kutta condition is selected because it can be shown that it is the only one that results in a stable flow pattern in the region of the trailing edge when the effects of viscosity are taken into account. The actual observations, of course, verify this physical reasoning and are the firm basis for the acknowledged correctness of the criterion. However, the observations do not explain the phenomenon; they only verify it. The explanation lies in consideration of the stability of the gross flow pattern.

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CONCLUSIONS

1. Visualization studies of adverse pressure gradient flows show that a regime of flow called transitory stall exists. This regime consists of a flow that is apparently unseparated but in which "spots of stall" appear and disappear randomly near the wall. This regime is not confined to a narrow range of flow conditions but instead is found in a wide variety of situations with the amount, size and duration of the stalls depending on the flow conditions.
2. Transitory stall appears to be the usual form of stall inception. That is, unless the flow is unusually uniform and free from disturbances or the stall is brought about by a sharp corner, stall appears to begin in a transient, three-dimensional manner as small transitory spots. This applies to both airfoils and diffusing passages.
3. The value of adverse pressure gradient and area ratio at which spots of stall are first found in a diffuser depends strongly on the amount of disturbance in the free stream. Larger disturbances cause small spots of stall to occur in the wall layers at lower values of adverse pressure gradient, and also increase the number of small stalls occurring for otherwise equal conditions. With very large disturbances in the free stream, small spots of stall can occur at very low values of adverse pressure gradient.
4. It is useful to distinguish four types of stall as follows:

   Let: w_b = backflow rate of stalled fluid over given section of wall

   w_e = steady rate at which stalled fluid can possibly be removed over the same wall section

Then: $w_b/w_e \ll 1$ Small transitory stall

$\left.\begin{array}{l} w_b/w_e < 1 \\ w_b/e_e \approx 1 \end{array}\right\}$ Large transitory stall

$w_b/w_e = 1$ Steady fully-developed stall

$w_b/w_e > 1$ Unsteady fully-developed stall (shedding wake)

In this classification, a fully-developed stall is one in which a backflow exists over an appreciable section of wall 100% of the time; a transitory stall is one in which a backflow exists over a given wall section only part of the time.

The flow behavior in the four types of stall listed is not entirely the same, and it is necessary to recognize the type of behavior occurring in order to rationalize behavior adequately. In particular, in the regimes of fully-developed stall it is apparently necessary to consider not only the value of w_b/w_e but also the effects of the interaction between the mainstream and the boundary layer and the stability of the entire flow pattern.

5. The regime of small transitory stall is characterized by small isolated spots of stall that occur in the wall layers. These stall spots move upstream a short distance, then leave the wall and are washed back downstream by the momentum forces of the mainstream. The existence of these small spots of stall even in relatively mild adverse pressure gradient flows explains the flow pulsations and excess losses so frequently found in diffusing passages. The spots of stall can be viewed as a means of providing the energy needed for the wall layers to move through the diffuser. The force analysis and the energy analysis both show that such energy is required in most cases. The force analysis indicates that spots of stall should be particularly prevalent just downstream from a point of

minimum pressure, and this has been found to be true. The energy analysis provides the minimum work that must be done in order to move a low energy particle through a diffuser. Since this work is in general done by irreversible shear forces, a means for roughly estimating losses, in addition to wall losses, can probably be obtained.

6. The regime of large transitory stall is a continuation of the regime of small transitory stall, and the distinction is one of convenience rather than kind. That is, as divergence angle (adverse pressure gradient) is increased, with inlet conditions and ratio of wall length to throat width held constant in a diffusing passage, the number and size of the spots of stall found increase. As this process is continued, the spots of stall finally reach a condition where they begin to interact and where they extend through the boundary layer into the main flow. Such a condition is called the regime of large transitory stall.

7. The regime of large transitory stall appears to break down into fully-developed stall by an unstable growth of a fixed spot of stall. The action is slow rather than fast; over a period of minutes, the entire flow pattern shifts from one of randomly occurring spots of stall to a fixed, fully-developed stall on one wall and apparently unstalled flow on the other.

8. In a diffuser followed by a plenum chamber, unsteady fully-developed stalls do not seem to occur. This is, apparently because the flow configuration is such that the fully-developed stall configuration is stable in the sense that the adjustment in w_b/w_e ratio brought on by changes in pressure forces is of a restoring nature. That is, a stable steady flow configuration always exists.

9. Conclusions 6, 7 and 8 above illustrate the nature of

the additional considerations beyond conventional boundary layer theory that must be introduced in order to rationalize the behavior of all types of stalls. Conclusions 1 and 2 also indicate that two-dimensional boundary layer theory by itself will probably not be sufficient to predict stall inception in at least many situations.

10. Conclusions 7 and 8 also show that the action of stall in passage flow is in some ways different from that found in external flow. Because of the introduction of stability considerations it is found that the presence of another wall can have an appreciable effect beyond that predicted from customary potential theory and boundary layer analysis, particularly for very high angles of divergence.

11. The basic action of vanes in diffusers apparently is related to conclusion 10; that is, the action of vanes of the type reported by Cochran and Kline (ref. 2) is not to alter the pressure distribution, but instead depends on a stabilizing effect under transient flow conditions. In particular, the vanes prevent the unstable breakdown from transitory to fully-developed stall mentioned in conclusion 7. This has been verified by showing that transitory stalls do occur in the wall layers both on the vanes and the side walls in vaned diffusers even when the overall performance is exceptionally high.

12. Under some circumstances, control of stalls can be achieved more readily by reduction of w_b than by increase in w_e. The important factor is the ratio. Explicit recognition of this criteria should be of considerable assistance in many design situations.

It is believed that the above conclusions should be of some assistance in the understanding and control of stall. The concepts developed are in agreement with the conclusions reached independently by Chapman, Kuehn and Larson (ref. 12) for the particular case of truly steady, fully-developed stall. The flow model found for transitory stall checks that found independently for other large Reynolds and Mach numbers by Scherrer and Lundell. The concepts and flow models developed also appear to give good qualitative rationalization of a wide variety of known but previously inexplicable phenomena. Despite these checks, considerable further work is needed both empirically and theoretically before final decisions can be reached on the ultimate usefulness and range of validity of the conclusions set forth above.

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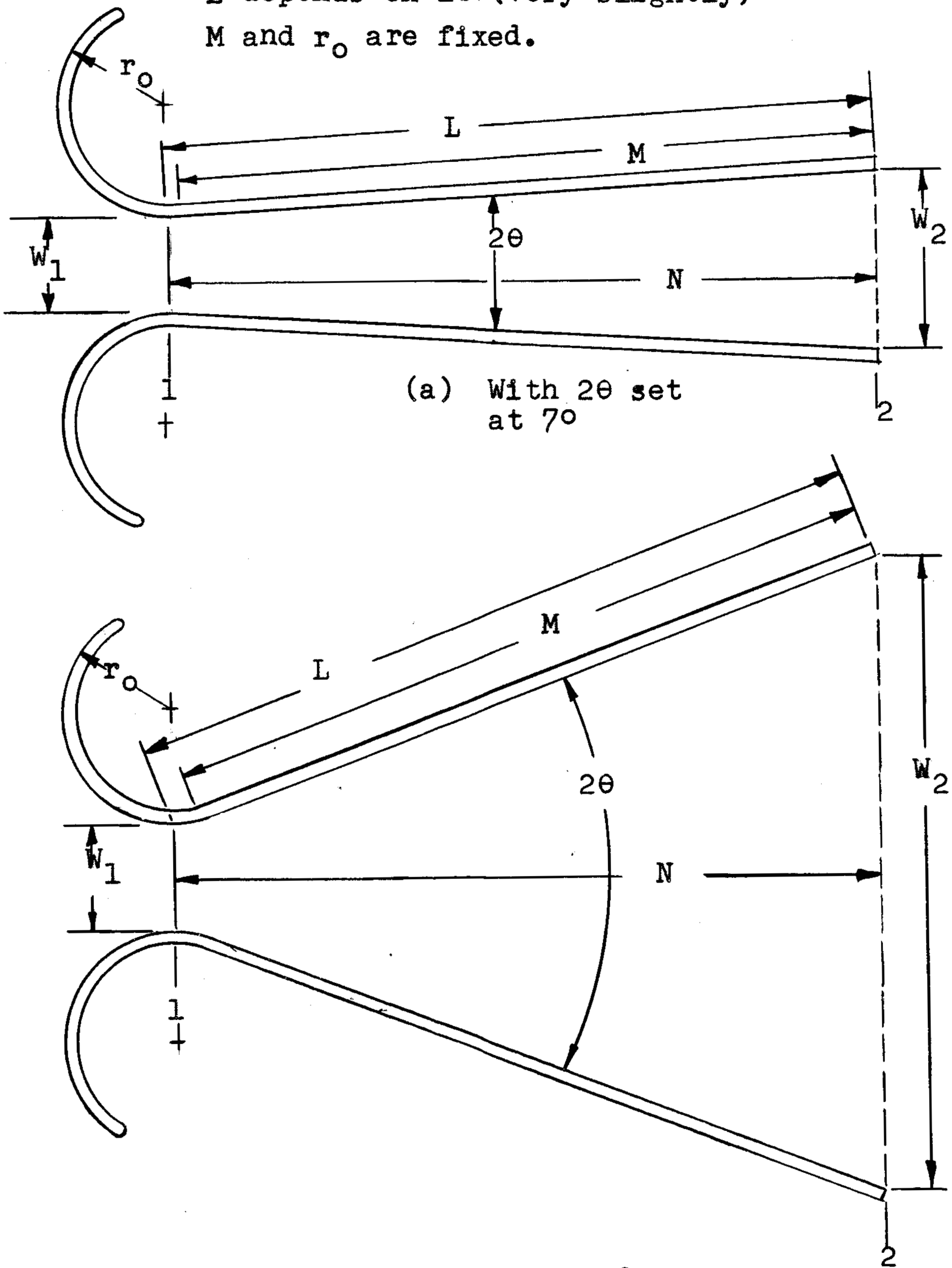
Note: W_1 and 2θ are independently variable.
N depends on 2θ and W_2 on 2θ and W_1.
L depends on 2θ (very slightly)
M and r_o are fixed.



(a) With 2θ set at 7°

(b) With 2θ set at 45°.

FIG. 1-- Geometry of Diffusers and Entrance Sections.

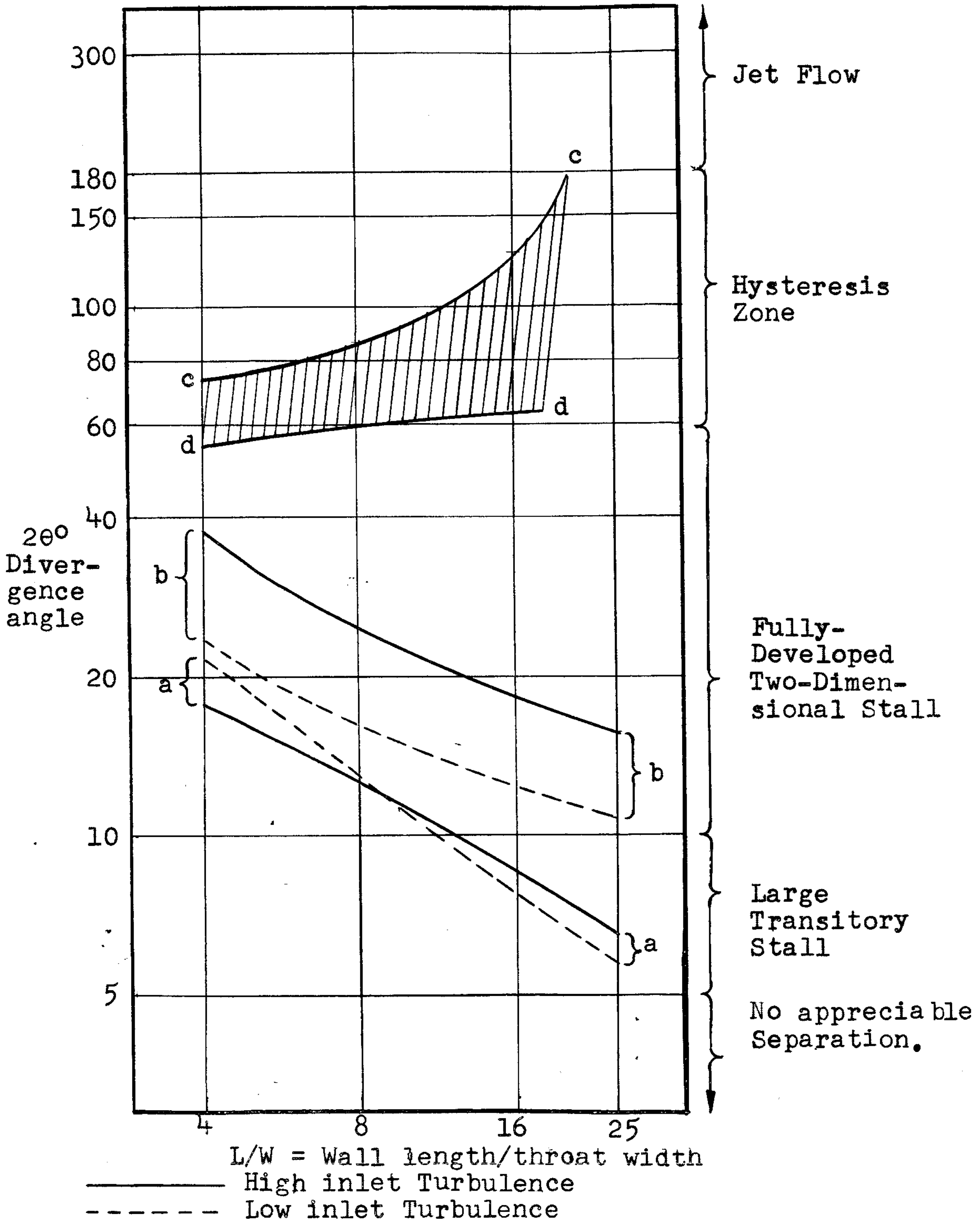
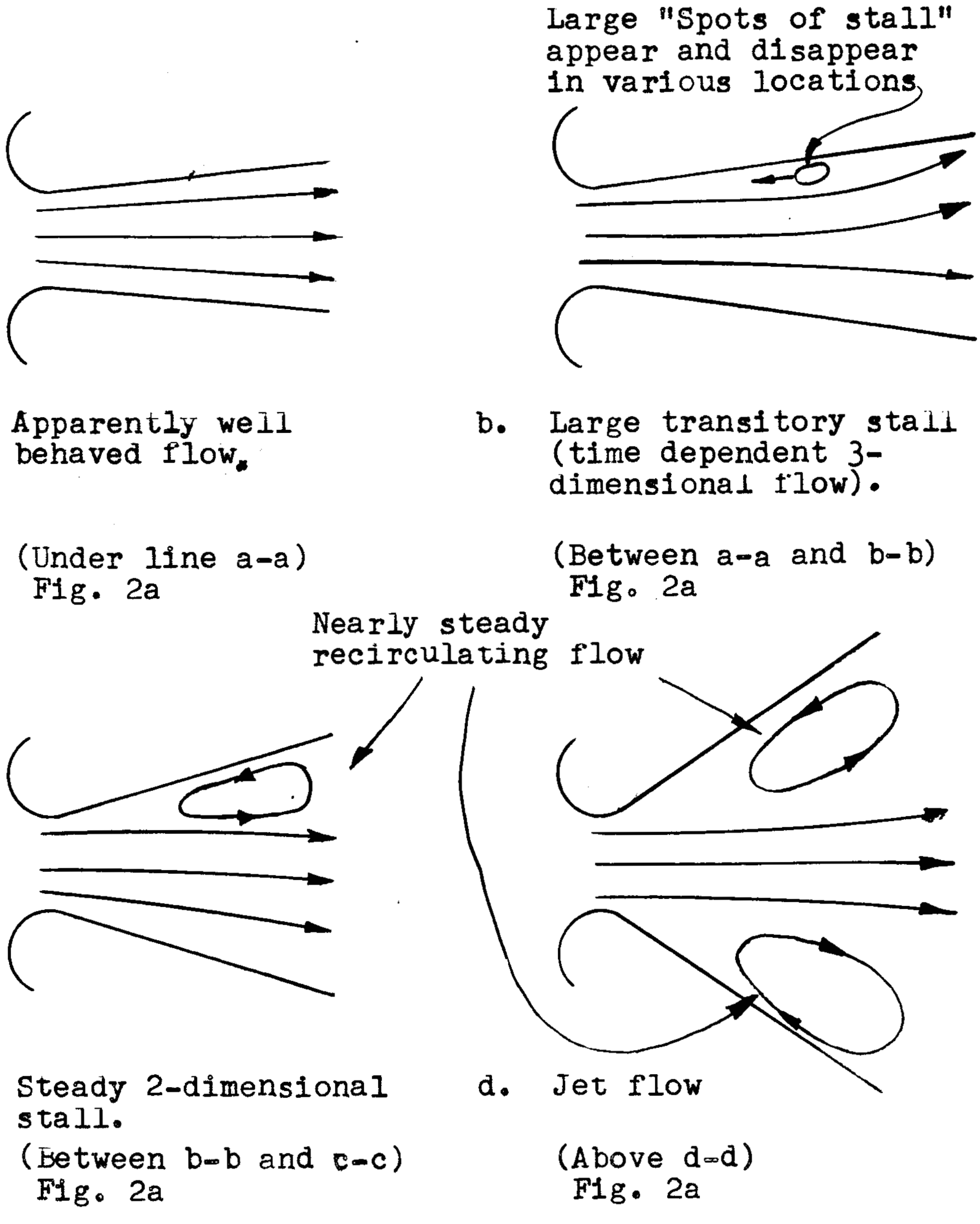


FIG. 2a--Zones of Diffuser Flow in terms of Governing Parameters for Fixed Mean Inlet Velocity Profile



a. Apparently well behaved flow.

(Under line a-a) Fig. 2a

b. Large transitory stall (time dependent 3-dimensional flow).

(Between a-a and b-b) Fig. 2a

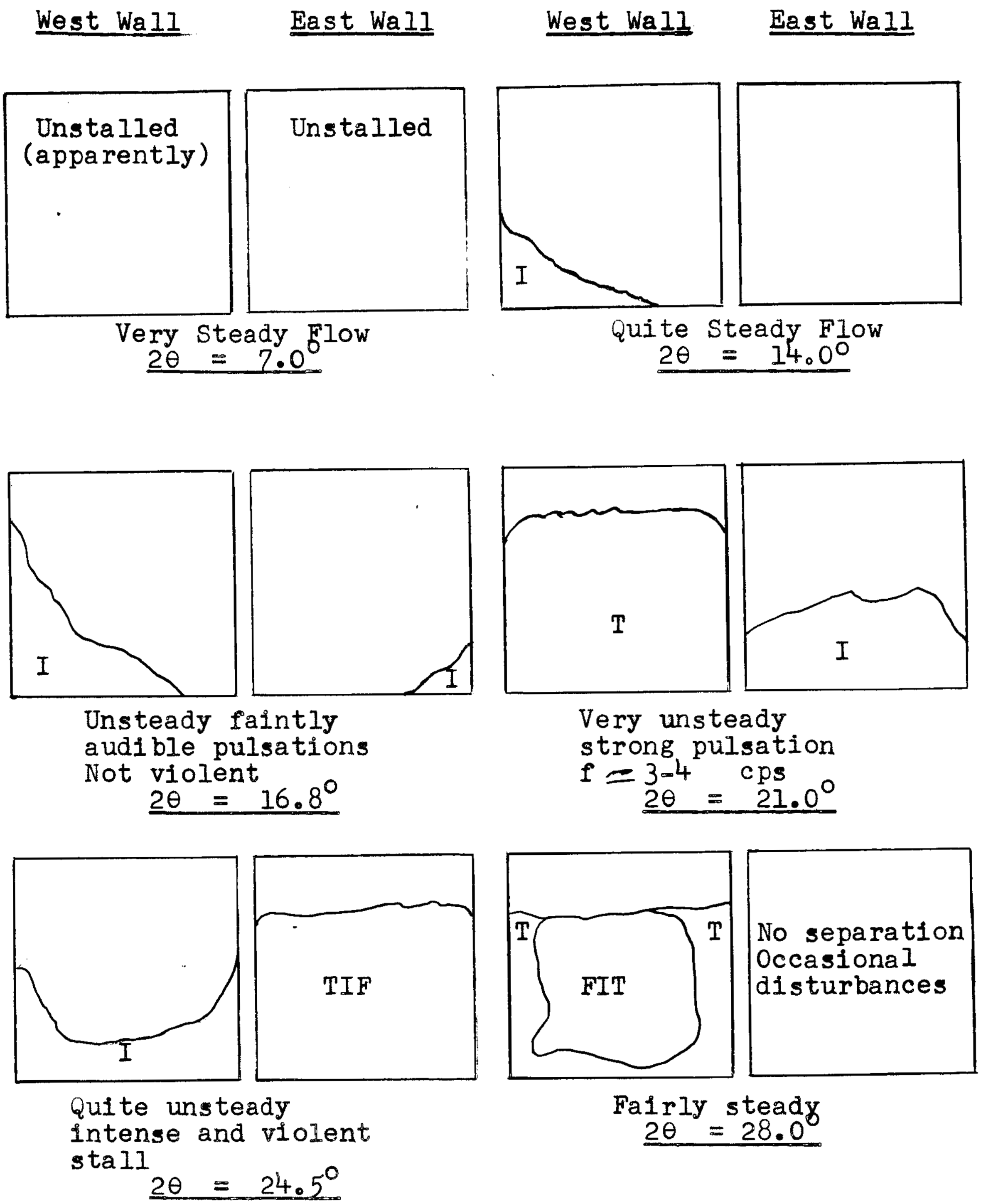
c. Steady 2-dimensional stall.

(Between b-b and c-c) Fig. 2a

d. Jet flow

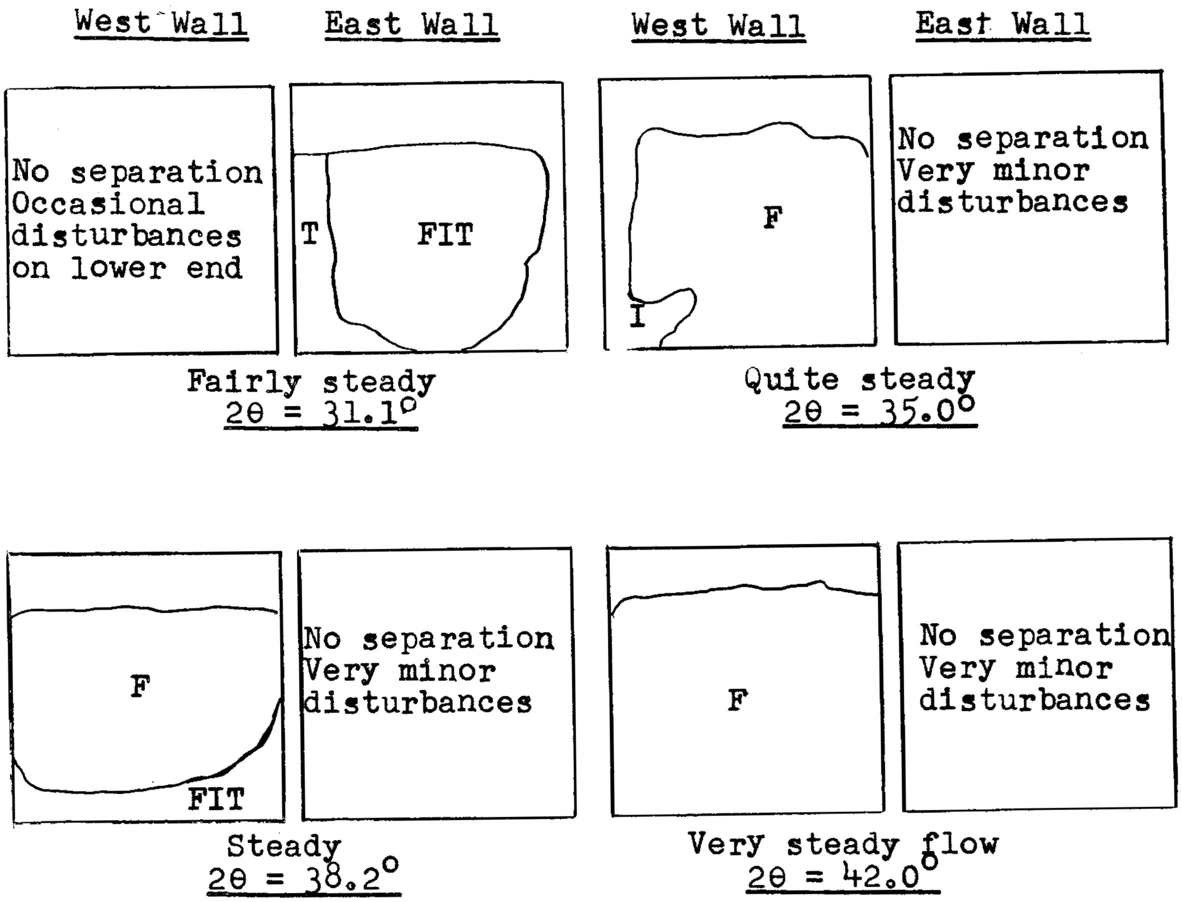
(Above d-d) Fig. 2a

FIG. 2b--Schematic diagrams of the regimes of diffuser flow.



Each pair of squares represents the diverging walls as seen looking out from inside the diffuser. I = intermittent transitory stall; T = transitory stall; TIF = transitory stall with intermittent fixed stall; FIT = fixed stall with intermittent transitory stall; F = stable fixed stall.
$R_{e_W} = 2.4 \times 10^5$, $L/W \simeq 8.0$, $W_1 = 3.00''$

FIG. 3--Sketches of Flow Patterns (See next page for higher values of 2θ).



Each pair of squares represents the diverging walls as seen looking out from inside the diffusers. I = intermittent transitory stall; T = transitory stall; TIF = transitory stall with intermittent fixed stall; FIT = fixed stall with intermittent transitory stall; F = stable fixed stall. $R_{e_w} = 2.4 \times 10^5$, $L/W \simeq 8.0$, $W_1 = 3.00''$

FIG. 3 cont.--Sketches of Flow Patterns

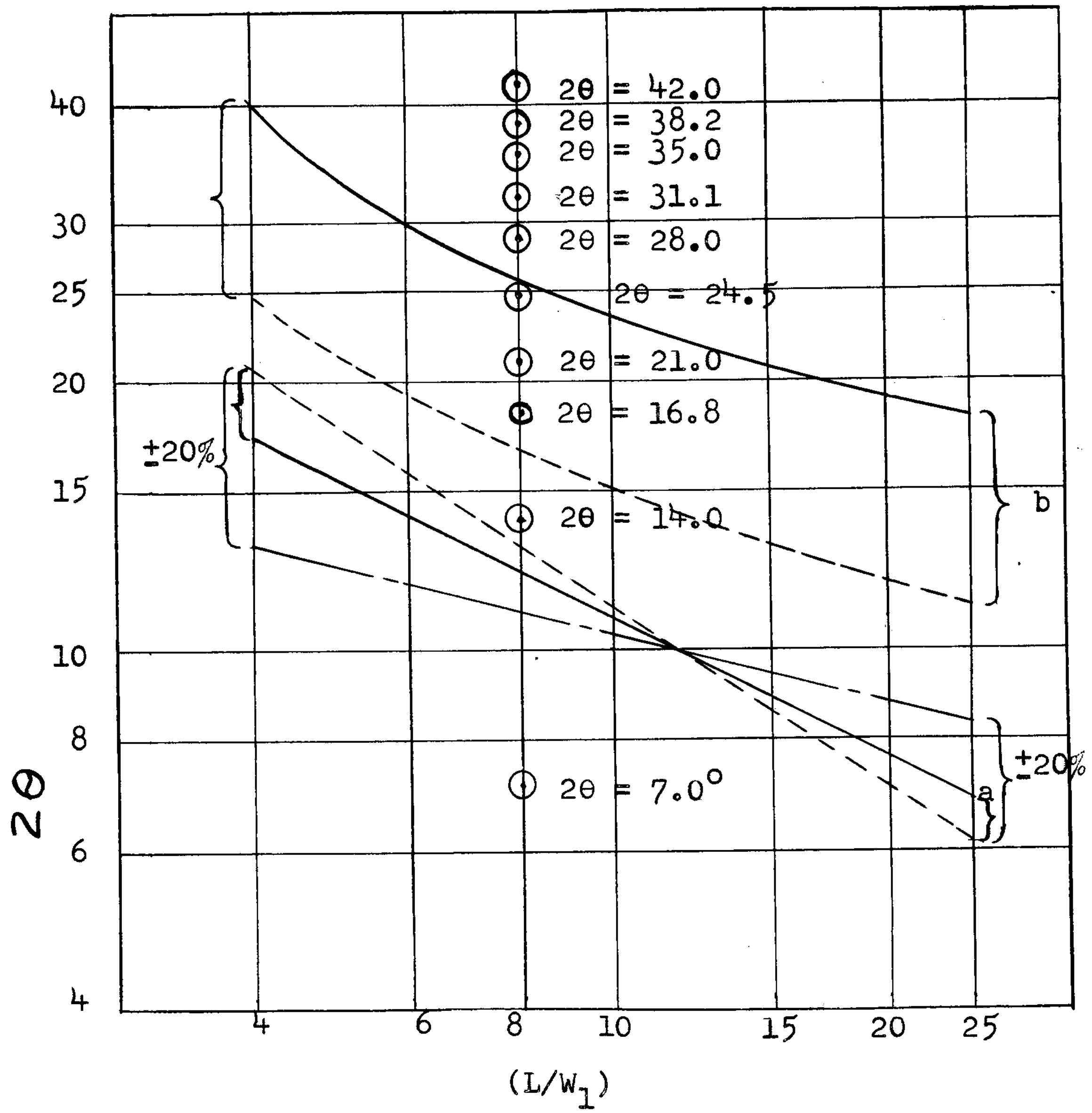


FIG. 4a--Correlation of flow patterns with regimes of diffuser flow.

2θ	Overall Flow Condition	Local Stall Condition
42.0	Very steady	2-D fixed
38.2	Steady	Nearly all fixed
35.0	Quite steady essentially complete 2-D	Mostly fixed
31.1	Fairly steady Nearly 2-D	Primarily fixed
28.0	Fairly steady nearly 2-D	Primarily fixed some transitory
24.5	Quite unsteady approaching 2-D	Primarily transitory some fixed
21.0	Firmly attached to one wall. Separation zones on other wall	Transitory
16.8	Unsteady but not violent	Transitory
14.0	Quite steady	Transitory
7.0	Very smooth minor end disturbance	No separation observable

Captions for Fig. 4a, 4b

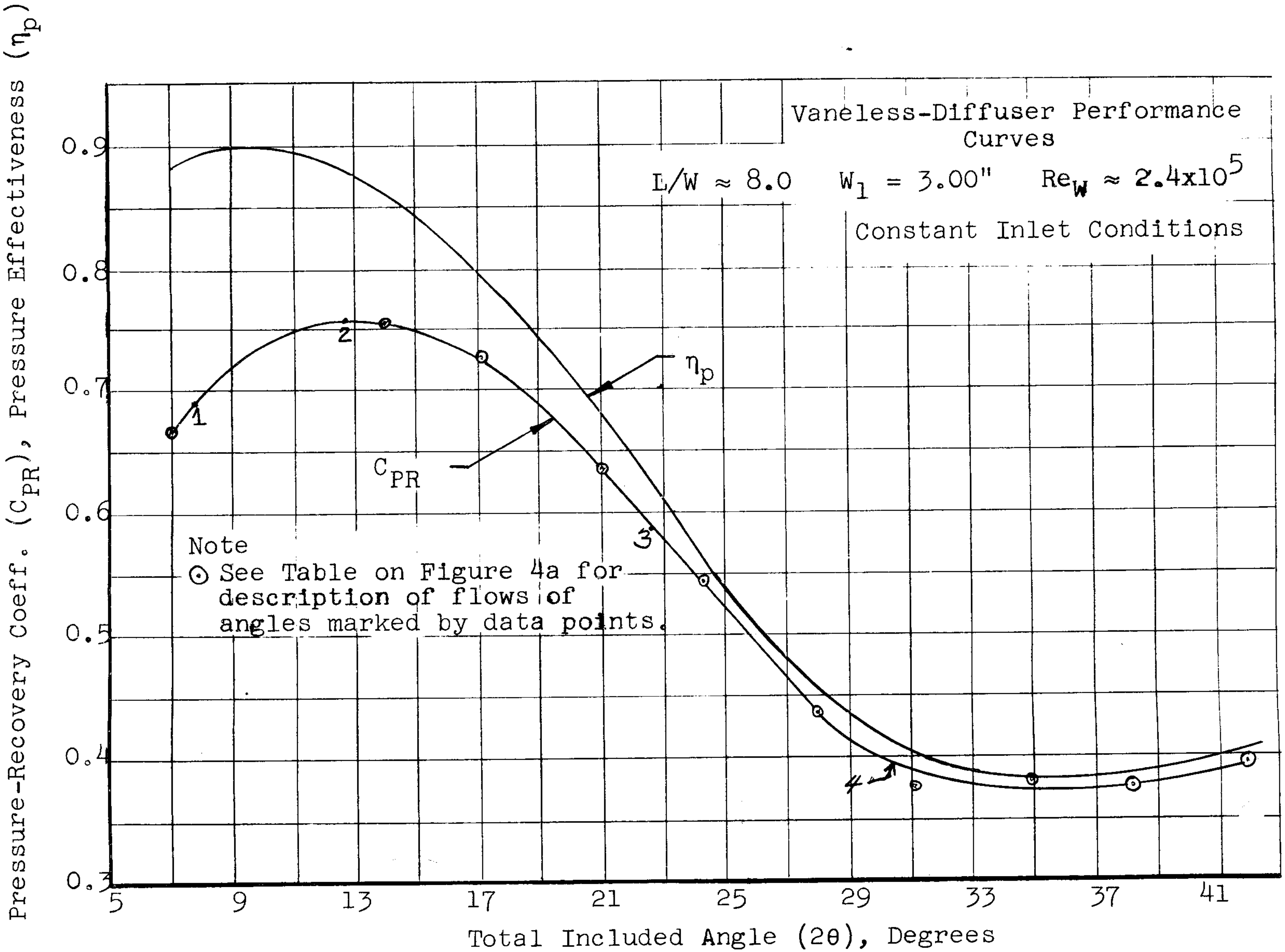
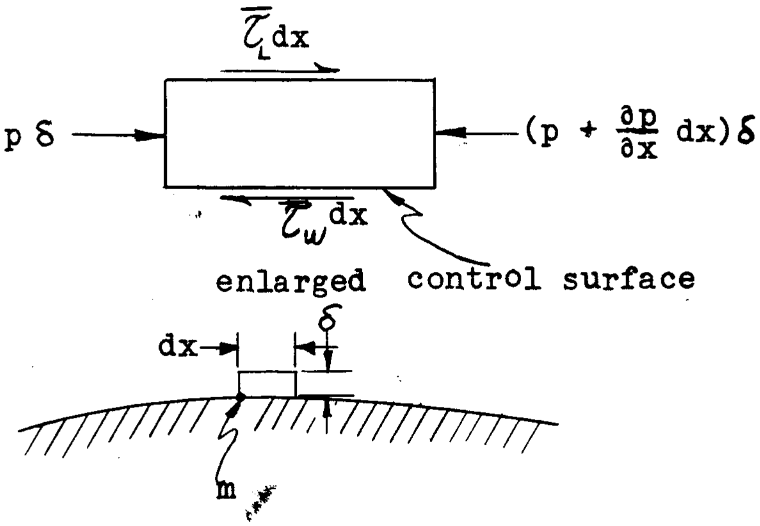
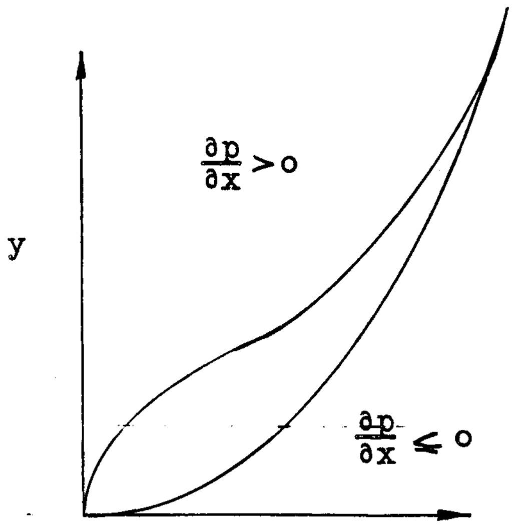

Vaneless-Diffuser Performance Curves
L/W ≈ 8.0 W_1 = 3.00" $Re_W \approx 2.4 \times 10^5$
Constant Inlet Conditions
η_p
C_{PR}
1
2
3
4
Note
⊙ See Table on Figure 4a for description of flows of angles marked by data points.
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Pressure-Recovery Coeff. (C_{PR}), Pressure Effectiveness (η_p)
5
9
13
17
21
25
29
33
37
41
Total Included Angle (2θ), Degrees

Figure 4b



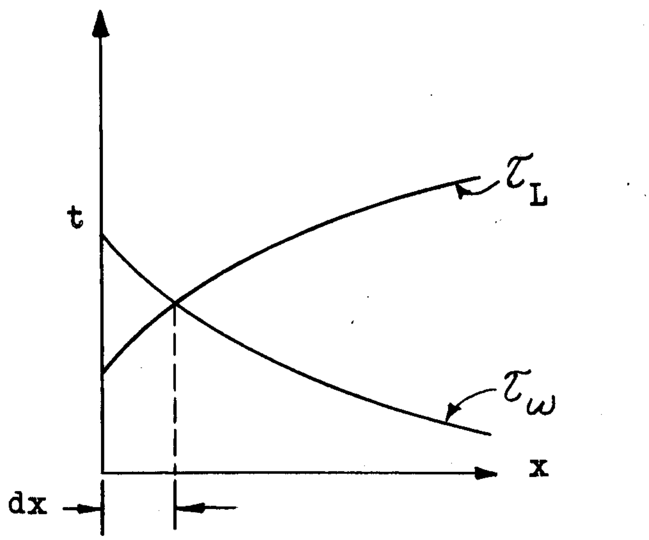
Control surface for force analysis of wall layers just downstream from point of minimum pressure

Fig. 5



Typical boundary layer velocity profiles for $\frac{\partial p}{\partial x} \leq o$ and $\frac{\partial p}{\partial x} > o$.

Fig. 6



Distribution of shear forces near wall just downstream from a point of minimum pressure.

Fig. 7

20 cc. Standard Yale Syringe

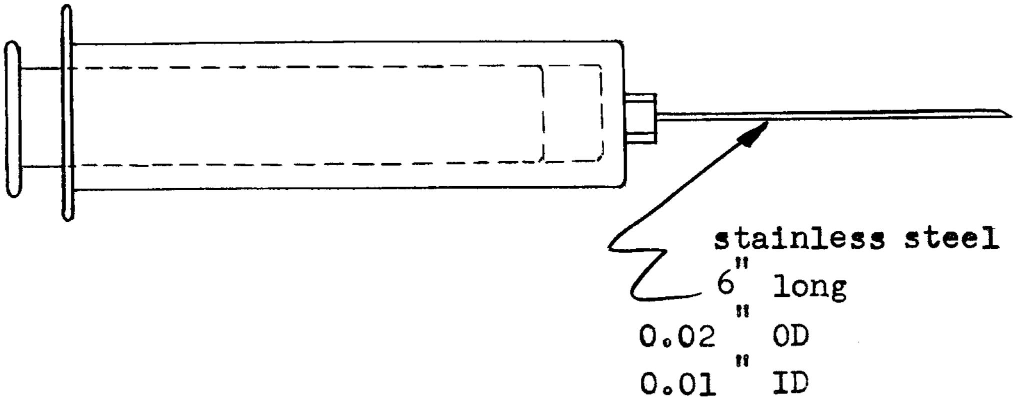
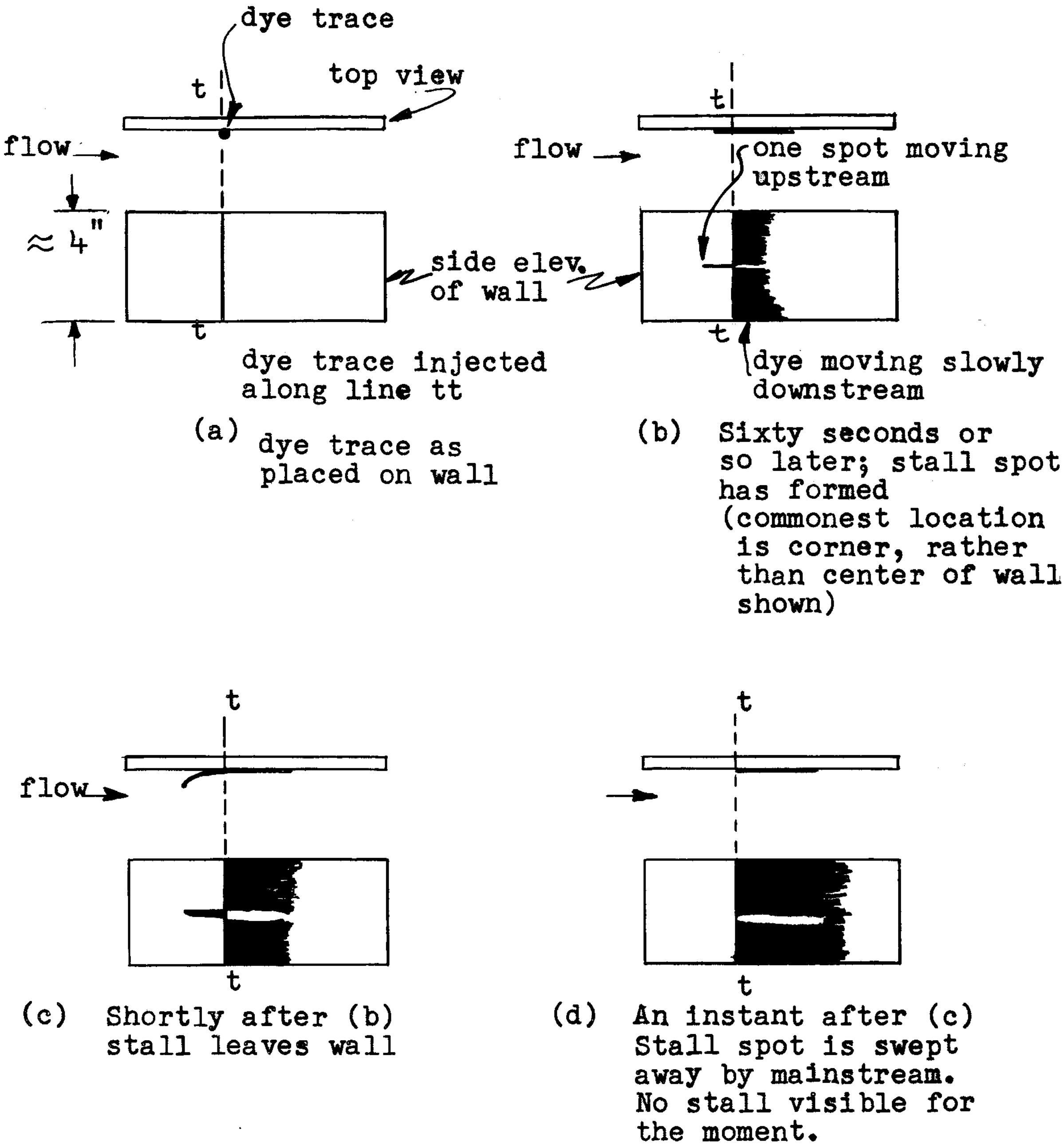
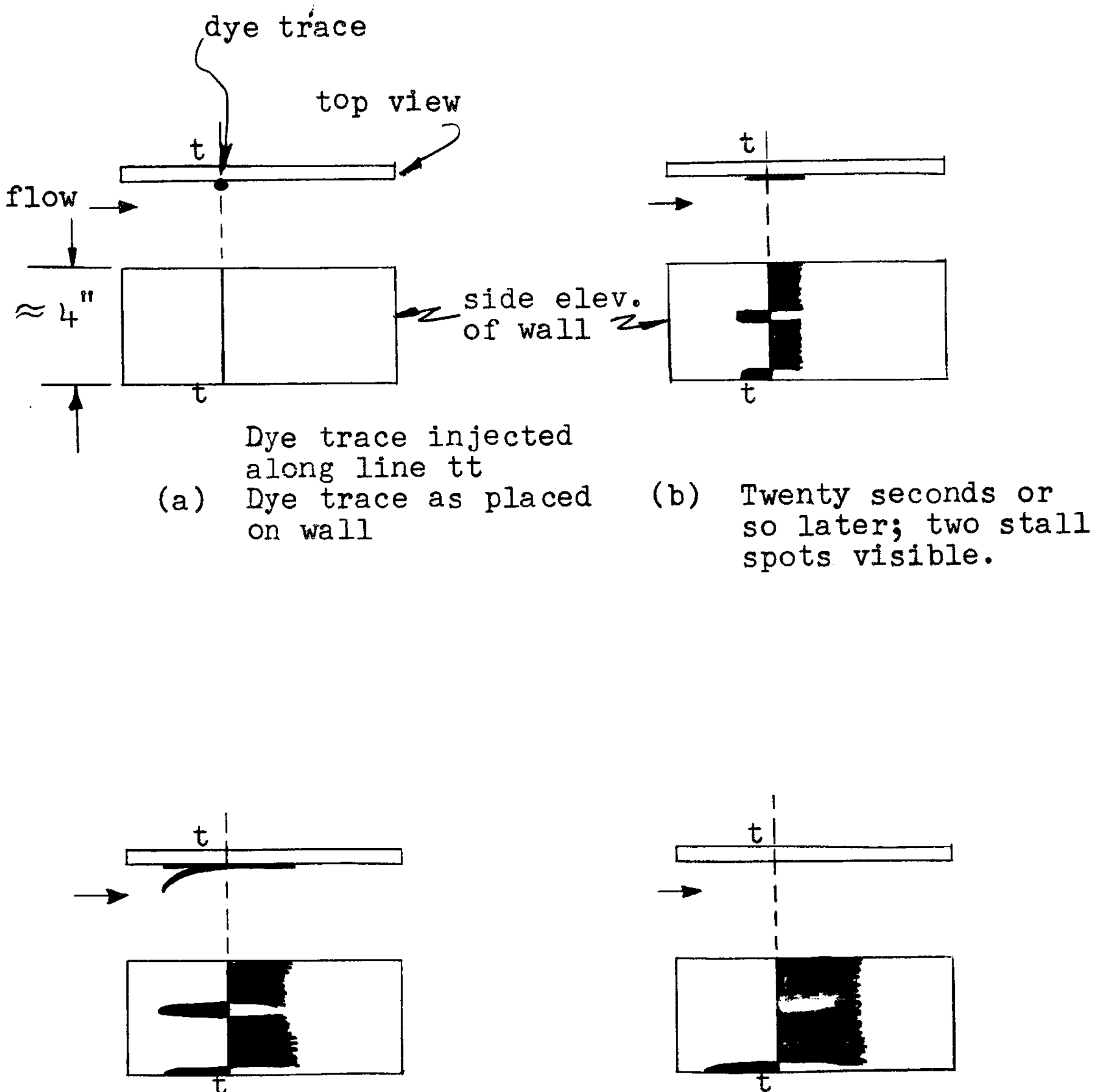


FIG. 8--Dye Injection Apparatus



Small transitory stall at approx. 2^{o} total divergence angle with very high free stream disturbances. Turbulent boundary layer; R_{e_x} of order of 2×10^5.

FIG. 9



(a) Dye trace as placed on wall

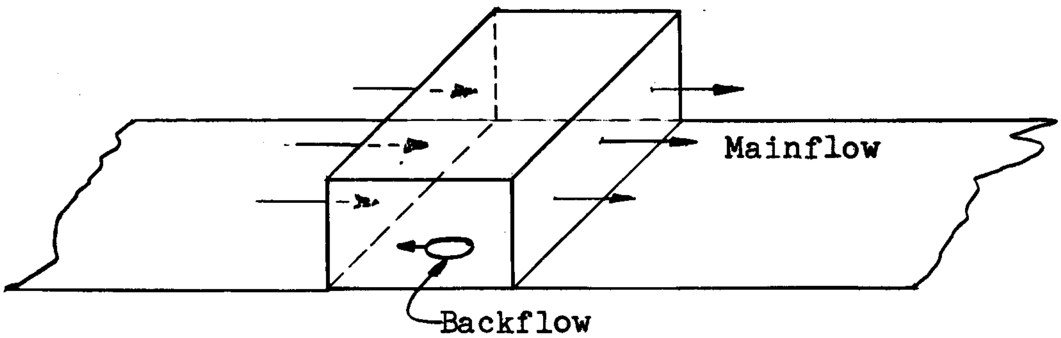
(b) Twenty seconds or so later; two stall spots visible.

(c) Shortly after (b) Upper stall leaves wall, corner stall still moving upstream.

(d) An instant after (c) upper stall swept away by mainstream (corner stall will probably shed next)

Small transitory stall at approximately 12° total divergence angle with very high free stream disturbances. Turbulent boundary layer; R_{e_x} of order of 2×10^5.

FIG. 10



Integral theorem of continuity:

$$\int_S \rho \, V \cdot dA = 0$$

FIG. 11--Control surface for continuity analysis of stall.

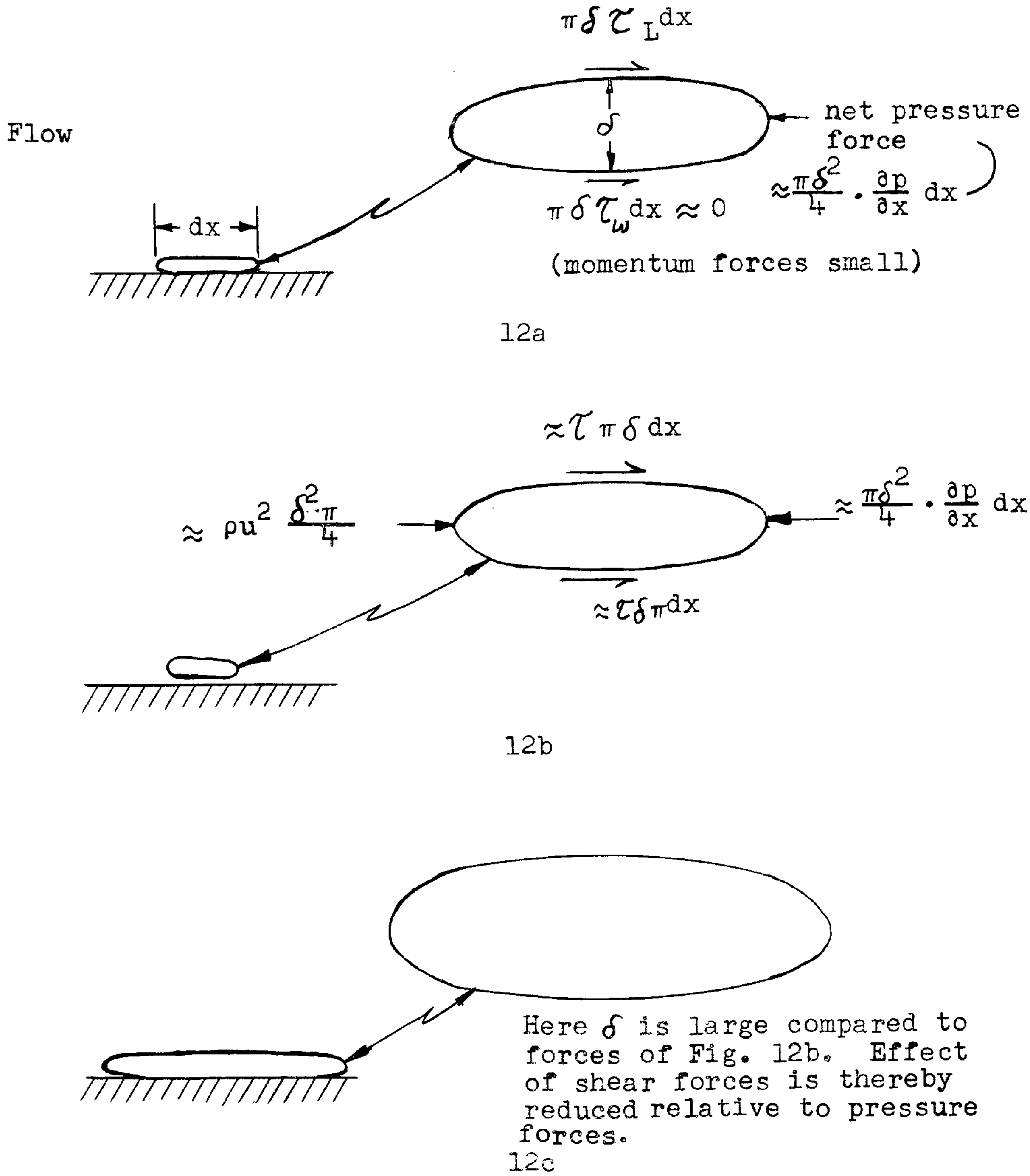
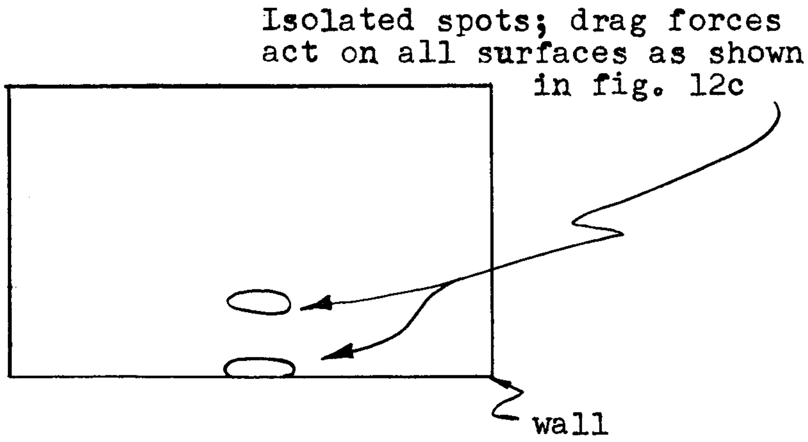
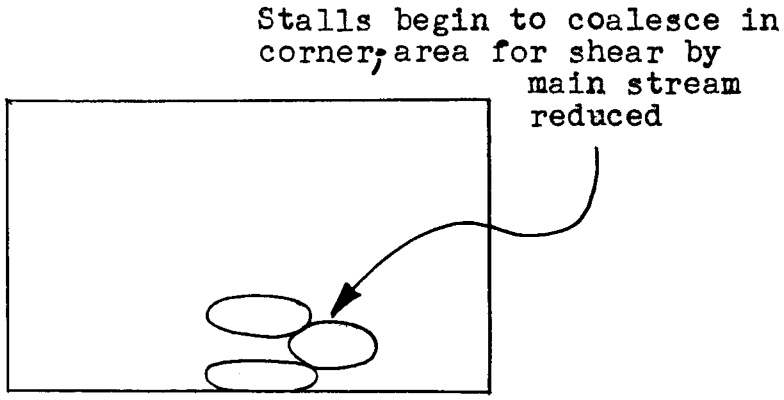


FIG. 12 --Qualitative nature of forces on spots of stall.

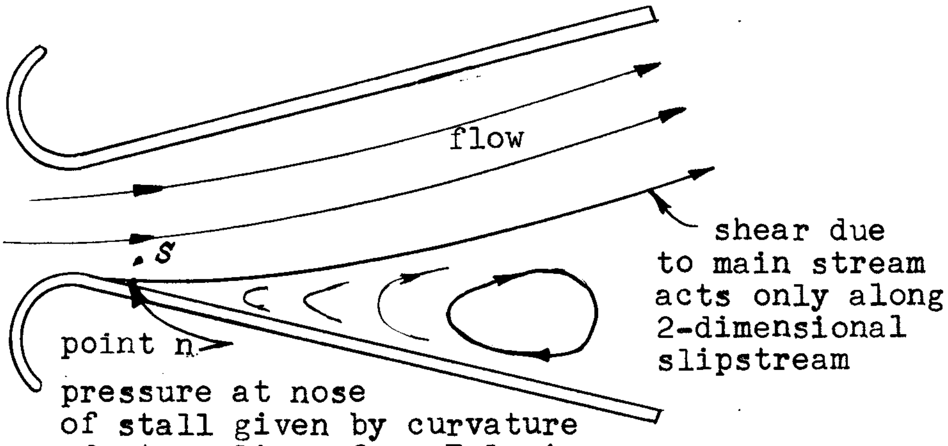


12d



12e

FIG. 12(cont'd).--Qualitative nature of forces on spots of stall.



pressure at nose of stall given by curvature of streamlines from Euler's Equation as:

$$p_n - p_s = \int_s^n \frac{\partial p}{\partial n}\, dn = \int_s^n \frac{\rho v^2}{R}\, dn$$

Hence average pressure near point n, p_n, is less than stagnation pressure of mainflow.

12f

FIG. 12 cont.--Qualitative Nature of Forces on Spots of stall.

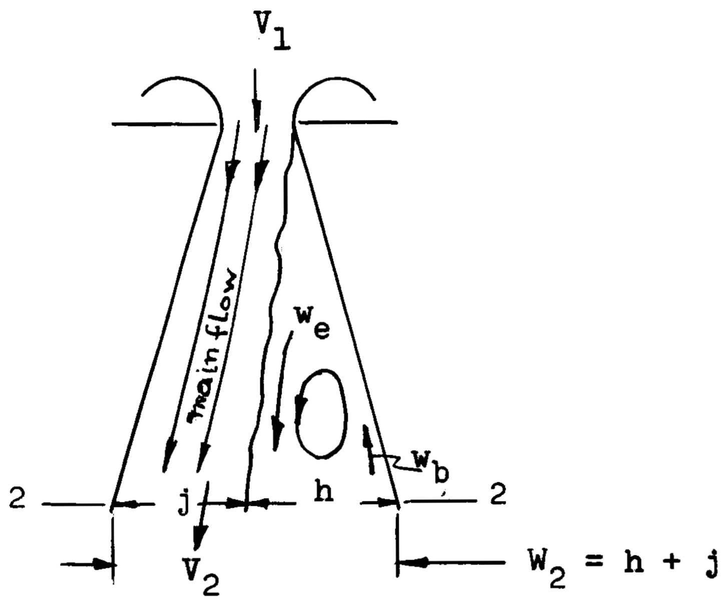


Fig. 13a

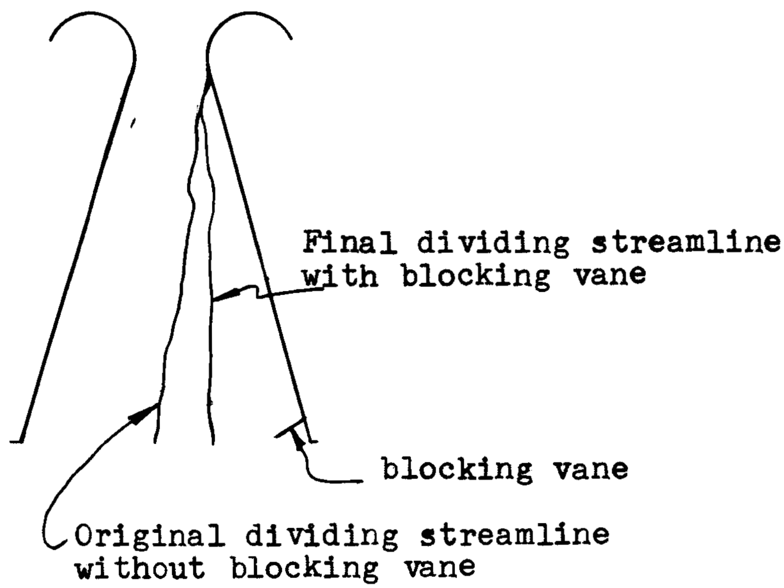
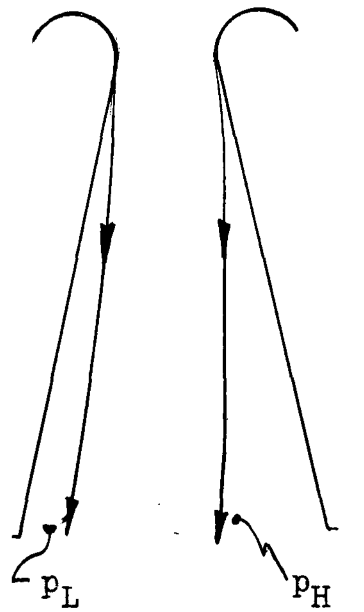


Fig. 13b--Effect on flow pattern of impeding backflow of stalled fluid in fully-developed stall.



Since equation of motion normal to streamline is:

$$\frac{\partial p}{\partial n} = \frac{\rho v^2}{R}$$

Pressures increases outward from center of curvature of streamlines, and $p_H > p_L$

FIG. 14--Unstable action of jet in confined channel.

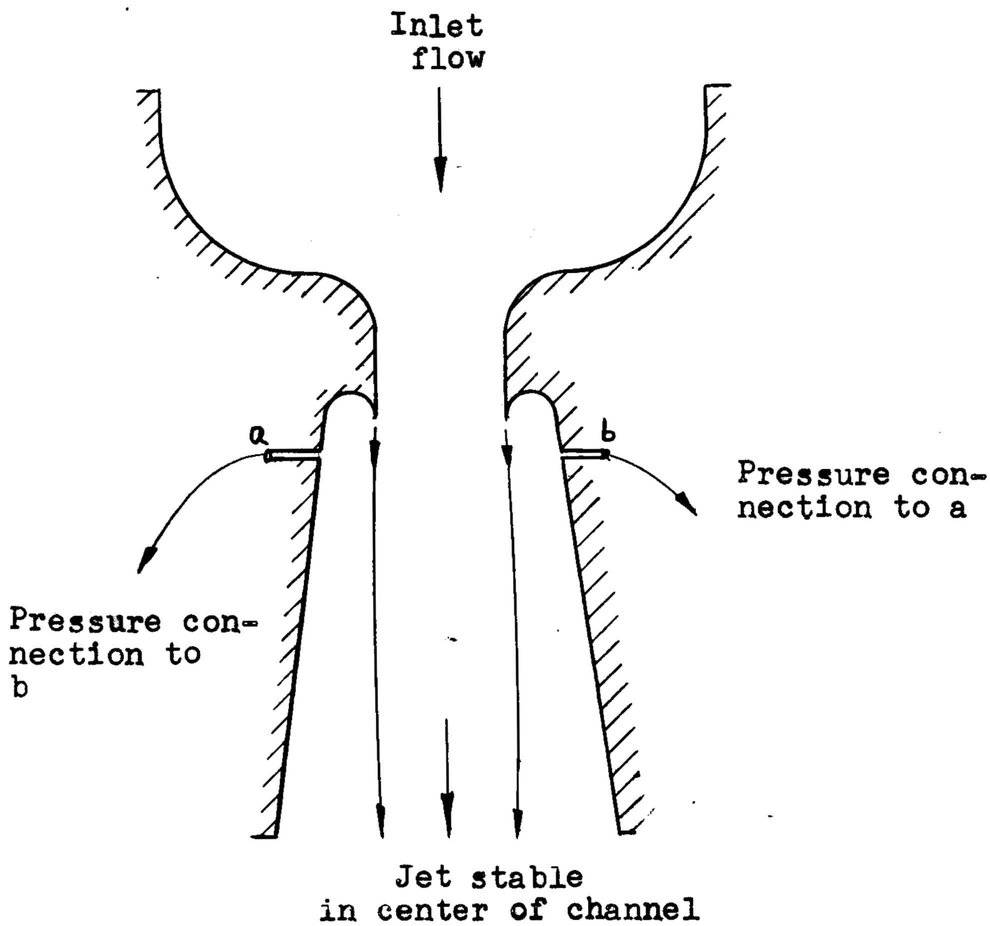
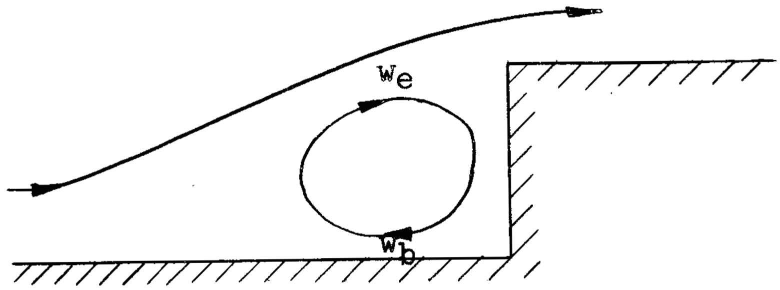
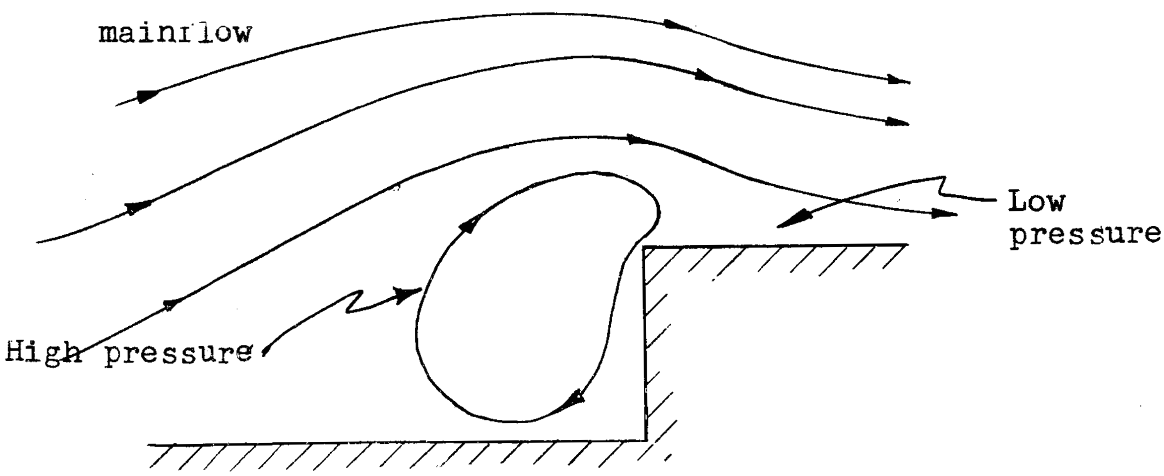


FIG. 15--Experiment stabilizing jet flow by equalizing pressures.



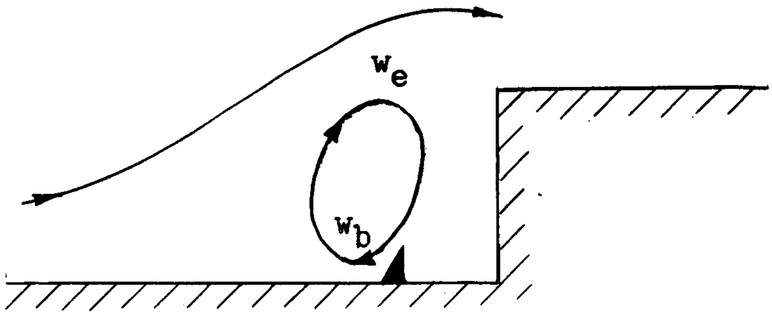
Stall in a contraction corner

16a



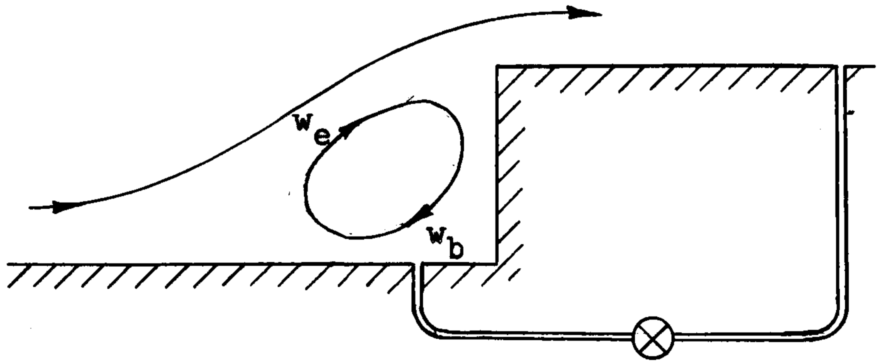
Unstable growth of contraction corner stall

16b



Possible stabilization of contraction corner stall by blocking w_b

16c



Possible stabilization of contraction corner stall by bleeding stall fluid

16d

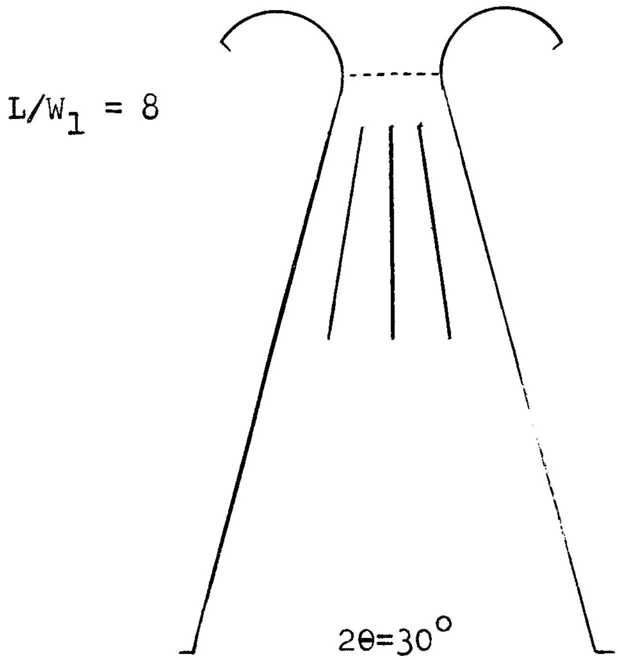
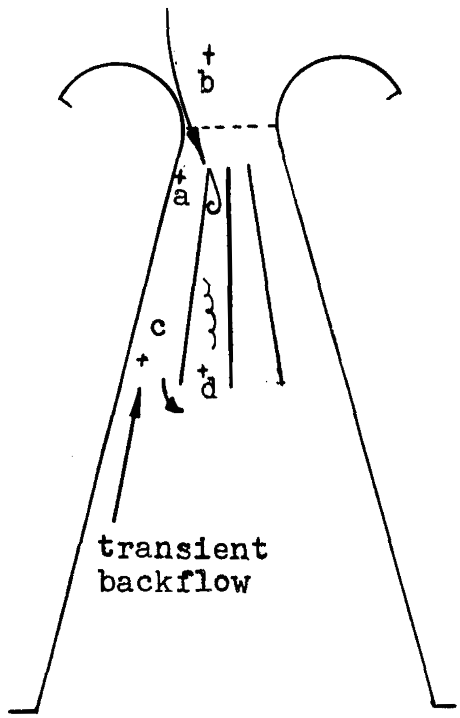


FIG. 17a--Typical short vane installation of type tested by Cochran and Kline.



1. $p_a > p_b$ due to Euler's N equation.
2. $p_c > p_d$ from known pressure distribution on stalled flat plate.

The tendency is to cause restoring flow because flow will go from c toward d. Without the vanes, such an effect would not occur.

FIG. 17b--Illustration of stabilizing action of vanes.

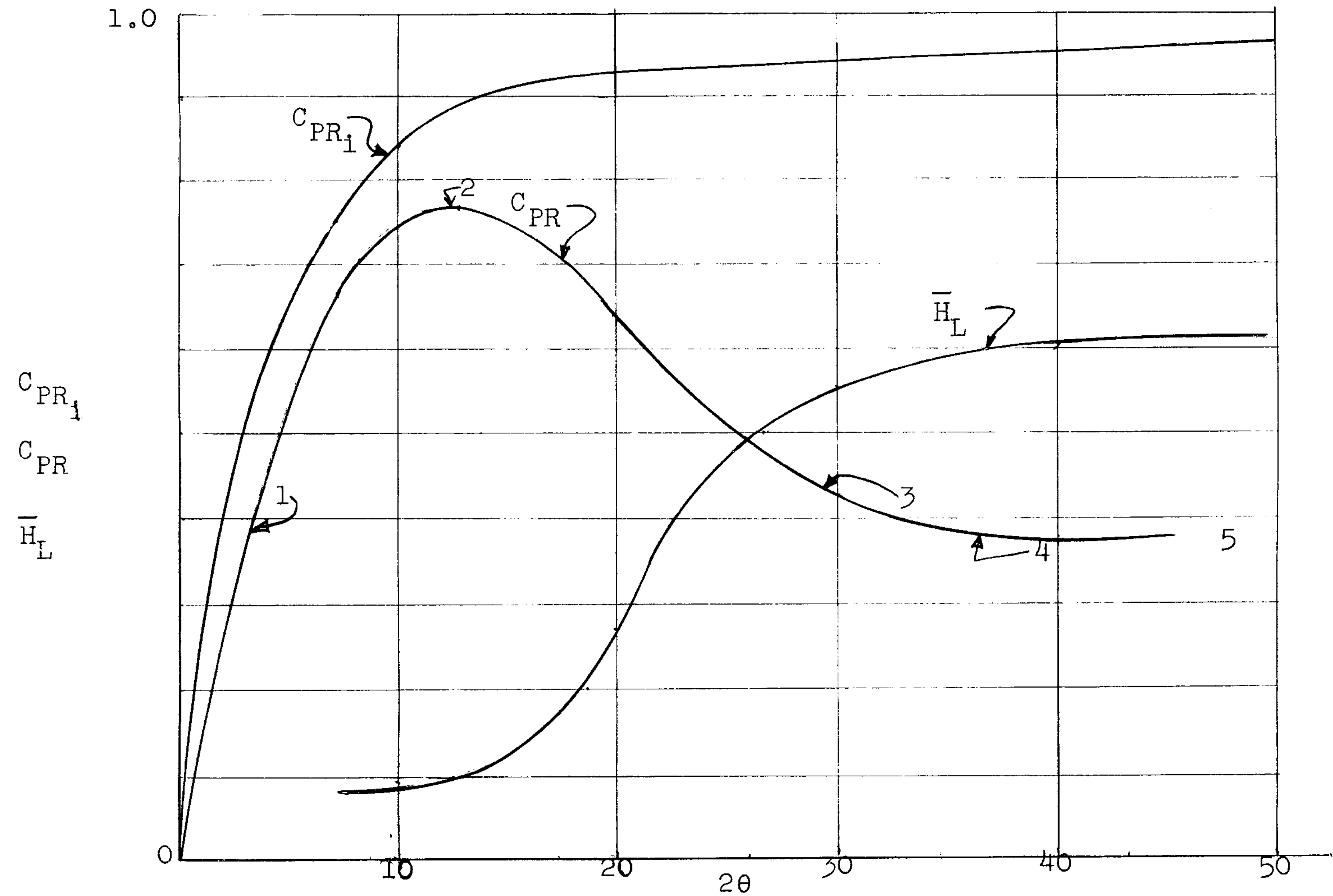


FIG. 18.--Actual pressure recovery, ideal pressure recovery, and head loss as a function of divergence angle for $L/W_1 = 8$.

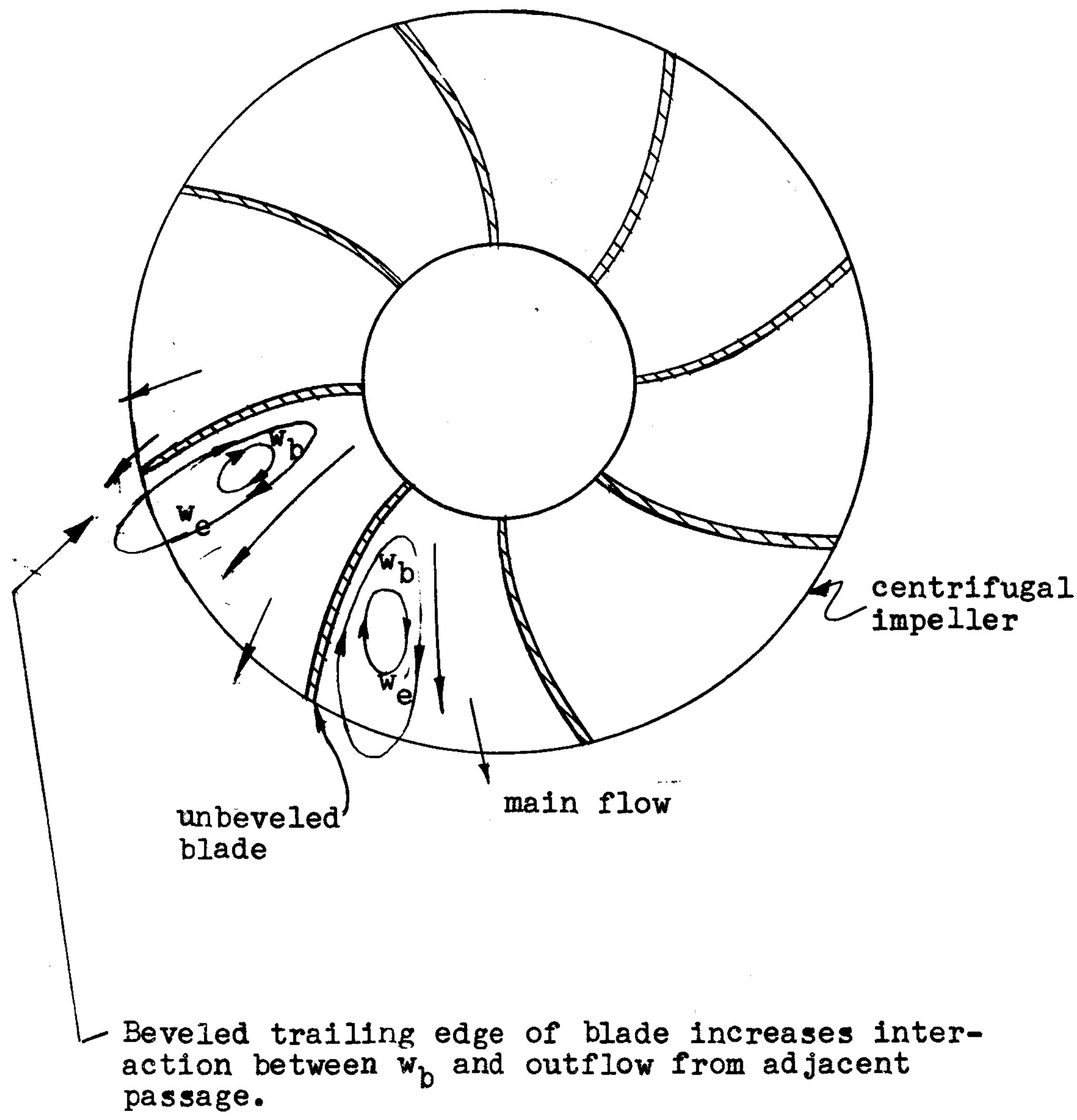
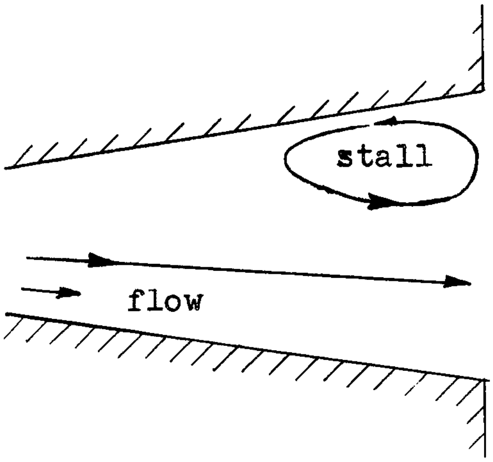
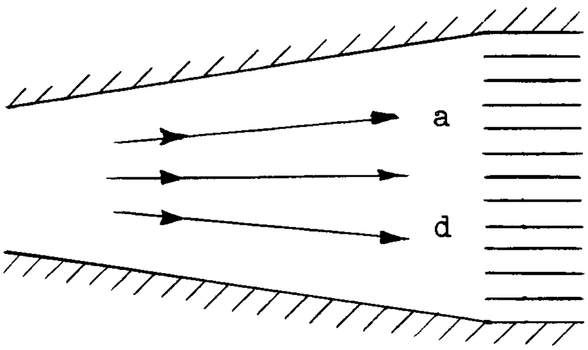


FIG. 19--Possible explanation of compressor blade underfiling.



Fully-developed passage stall.

Fig. 20a



Tube bundle

Passage of Fig. 19a filled by alteration of downstream geometry.

Fig. 20b

FURTHER COMMENT AND ERRATA (ON REPORT MD-1 ENTITLED "SOME NEW MECHANISMS AND CONCEPTIONS OF STALL INCLUDING THE BEHAVIOR OF VANED AND UNVANED DIFFUSERS" 15 MARCH 1957, STANFORD UNIVERSITY).

Several readers of this report, which was prepared as a report of progress and for comment, have pointed out deficiencies in the discussion given. Since these deficiencies do not affect the utility of the basic concepts developed, the following comments are added to minimize possible misconceptions until such a time as the entire report can be rewritten.

ERRATA:

1. Page 51. Equation (10) is missing a parenthesis. It should read:

$$C_{pr} = 1 - \left(\frac{W_1}{W_2 - h}\right)^2$$

2. Page 76, last line. Reference should be to figure 20a instead of 19a and the words "as indicated by the dotted lines" should be deleted.

3. Page 112. Reference in title of figure 20b should be to figure 20a. Also the position of the letter "d" should be interchanged with "a" and "b" should be interchanged with "c".

4. Page 77, line 7. The last letter should be "d" instead of "c".

5. In all of the discussions of the flow patterns observed, particularly in regard to fully-developed stall, it should be borne in mind that no stalled flow is completely steady; some fluctuations always occur. The descriptions given attempt to incorporate the major patterns to develop adequate approximate flow models, but fluctuations that appeared to be second order, compared to the main patterns of flow, have been omitted from the present discussions for the most part. This is deemed appropriate, but should have been stated more explicitly.

6. In the definitions of w_b and w_e it is of utmost importance to note that w_b is the ACTUAL backflow rate existing, but w_e, on the other hand, is defined in terms of the steady capability of the stream to remove backflow under the conditions existing at the time in question; this may or may not be the actual rate of removal at any given instant. This is in agreement with the definitions given, but apparently they are not explicit enough since some readers have not comprehended the distinction and have thereby found it difficult to understand virtually everything that follows.

The above comments are due primarily to Dr. D. L. Cochran to whom the author wishes to express his appreciation. The following two errors were noted by Dr. R. C. Dean, Jr. to whom the author is much indebted for several helpful discussions as well as the following comments. Dr. Dean is also primarily responsible for the discussion on the centrifugal impeller although an acknowledgement on this point was omitted in the draft due to an oversight.

7. The discussion of the forces at a point of minimum pressure is not conclusive. The conclusion reached is verified by observation, but a possible solution of the equations involving upstream acceleration of the wall layers in a two-dimensional fashion has been overlooked. In the final report other data will be substituted for this particular argument.

8. The rationalization of the known behavior of the free jet between channel walls is probably wrong. The facts are correct, since they are based on observation, but the explanation is not adequate since similar reasoning looking downstream yields the opposite conclusion. As Dr. Dean has pointed out, the effect is probably simply associated with the width of the effective jet pump formed by the slipstream of the separated flow. This point needs further consideration.

S. J. Kline
6/24/57